

Collateral Shortages and Intermediation Networks*

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Abstract

This paper argues that in the presence of trading frictions and agency problems, the inter-bank market may be overly fragile, in the sense that small changes in the liquidity of assets used as collateral may lead to large swings in haircuts and a potential credit freeze. Our results highlight that the financial system's intermediation capacity crucially depends on the distribution of collateral among financial institution rather than on its aggregate amount. In particular, we show that bottlenecks might endogenously spring up and impair credit relationships. These results thus provide a novel explanation for the collapse in the secured lending market during the recent crisis.

Keywords: financial networks, intermediation capacity, secured lending, collateral.

JEL Classification: G01, G20, D85.

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1 Introduction

In the pre-crisis period, financial markets witnessed a growing reliance on short-term funds raised in wholesale markets. In particular, there was a dramatic rise in the use of sales and repurchase (repo) agreements to fund longer-term investment opportunities or to finance inventories of securities held for market-making purposes. Given that such funding opportunities were secured by collateral, they were mostly considered to be safe. Since the crisis of 2007–8, however, the repo and the asset-backed commercial paper (ABCP) markets have been viewed as one of the potential sources of fragility in the financial system. For instance, as documented by [Gorton and Metrick \(2012a,b\)](#), concerns about the risk and liquidity of the collateral at the onset of the crisis led to a dramatic rise in inter-dealer bilateral repo haircuts, a run in the repo market, and a collapse in short-term (secured) lending. Similarly, [Covitz, Liang, and Suarez \(2013\)](#) show that ABCP outstanding began to plummet in the summer of 2007, as one-third of ABCP programs experienced a run due to mounting concerns about the default risk of subprime mortgages.

In this paper, we argue that in the presence of moral hazard and trade frictions among financial intermediaries, the interbank market would be overly fragile, in the sense that small changes in the liquidity of assets used as collateral may lead to large swings in haircuts, a significant drop in the financial system’s intermediation capacity, and a potential credit freeze. Such fragility emerges even if banks rely on secured lending contracts, such as repurchase agreements. This is due to the fact that in the presence of trade frictions, moral hazard problems would cumulate over intermediation chains, creating a channel over which small shocks to the collateral liquidity can propagate and amplify, leading to large aggregate effects. Crucially, we also show that the financial system’s intermediation capacity is highly sensitive to how collateralizable assets are distributed throughout the system.

Our results contribute to a recent debate among economists and policymakers about the possibility that financial markets may be experiencing a collateral shortage. On the one hand, many economists argue that the new regulatory environment and the increased concerns about counterparty risk — which has led market participants to seek safety by demanding more and better collateral to support their transactions — have resulted in a dramatic shortage of collateral ([Anderson and Jøeveer, 2014](#); [Singh, 2011](#)). On the other hand, others have been more skeptical of this view, noting that the market may be experiencing only temporary supply-demand imbalances as the aggregate amount of securities held by financial institutions has not changed significantly ([Juks, 2012](#)). We contribute to this debate by showing that, since the distribution of collateral (as opposed to its aggregate amount) determines the system’s intermediation capacity, bottlenecks might endogenously spring up and impair credit relationships. In other words, judging the resilience of the secured lending market by the aggregate amount of securities misses an important source of fragility, namely the distribution of collateral among intermediaries.

To capture the above ideas concretely, we focus on an economy consisting of n financial institutions (henceforth, banks) some of which are endowed with excess capital, while others have access

to profitable investment opportunities. However, due to the presence of trade frictions — which may emerge due to pairwise commitment problems, search frictions, adverse selection, or the absence of long-term interbank relationships — not all banks are able to trade directly with one another, making intermediation necessary for the realization of the gains from trade. Formally, we capture the presence of such trade frictions by the means of a *financial intermediation network*, which determines the set of bank-pairs that can enter into bilateral contracts with one another.

In addition to trade frictions, financial intermediation in our model is subject to another key friction: any of the intermediaries can divert the funds to invest in potentially riskier, inefficient projects, making all bilateral transactions subject to moral hazard. In particular, we assume that the investment decisions of the intermediaries are not contractible, and that lenders cannot write contracts that are contingent on the intricate patterns of trade within the financial system. Rather, they are restricted to writing simple contracts that only specify the interest rate and the haircut they are willing to charge the borrower. We believe this captures an important feature of reality as lenders are not able to scrutinize every investment made by their counterparties in real time and this exposes them to risk. Intuitively, in our model counterparty risk emerges because banks might engage in excessively risky activities and default on their obligations.

To highlight the crucial role played in our analysis by the distribution of collateral and its illiquidity, we start our analysis by showing that as long as there is an abundance of liquid assets to be employed as collateral, bilateral secured lending contracts can overcome all moral hazard frictions within the financial network. More specifically, we show that, despite the absence of contingent contracts, the investment choices of all borrowers in the network can be influenced by increasing the amount of collateral that is required to be posted with the lender. The haircuts thus essentially serve as an instrument that can restore efficiency when borrowers have an incentive to take excessive risk. This result thus echoes the earlier findings by [Stiglitz and Weiss \(1981\)](#) and [Bester \(1985, 1987\)](#) who show that collateralized lending can overcome moral hazard problems. Our paper, however, show that simple secured lending contracts not only can overcome moral hazard problems in bilateral lending agreements, but can also discipline the investment choices of *all* intermediaries in the network — including the ones with whom they are not directly contracting.

Even though secured lending contracts can restore first-best efficient outcomes when banks have large endowments of assets that can be used as collateral, the same is no longer true when collateral is scarce. Such a scarcity means that lenders may no longer be able to overcome the moral hazard problems, thus leading to an upper bound on the volume of capital that can be efficiently intermediated through the system. Our results provide a characterization of this upper bound — which we refer to as the financial system's *intermediation capacity* — in terms of three key characteristics of the collateral: its *liquidity*, *volatility* and *availability*. *Liquidity* refers to the discount that a lender faces when selling it on the secondary market due to the borrower defaulting on its obligations, e.g. treasuries will be more valuable as collateral than corporate bonds due to the ease of resell. *Volatility* captures the riskiness of the collateral, while *availability* measures the amount of collateral that each borrower can post with their lenders.

Since moral hazard problems are more severe in longer chains, as more intermediaries can divert the funds towards risky and inefficient projects, idiosyncratic shocks may be amplified along the chain of intermediaries. In particular, we show that small idiosyncratic shocks (say, to the expected return on the efficient project or the liquidity of the collateral posted by bank i) can lead to a sharp increase in haircuts, and as a result, large drops in the financial system's intermediation capacity, which may even lead to a complete credit freeze. Interestingly, our results also show that pledging a low quality collateral by a peripheral intermediary can affect yields and haircuts of all market participants, even those to whom the intermediary is not directly contracting. This observation highlights the fact that the juxtaposition of trade frictions and agency problems can lead to the build up of systemic risk by amplifying the effect of small idiosyncratic shocks. Such events, which are reminiscent of repo runs and the severe contraction in the ABCP market observed during the crisis, arise not due to the intermediaries' need for funding liquidity or shocks to expectations about future returns (as argued by [Brunnermeier and Pedersen \(2009\)](#) and [Martin, Skeie, and von Thadden \(2014\)](#), among others), but rather as a response to the build up of moral hazard frictions over intermediation chains.

Using this characterization, we then show that the distribution of collateralizable assets among different financial institutions also plays a first-order role in determining the equilibrium allocation of intermediation rents, and hence, the incentives of different institutions to avoid diverting the funds towards inefficient projects. Specifically, in an intermediation chain some of the intermediaries' collateral constraints may endogenously bind, that is, some financial institutions may not have enough assets to pledge as collateral to intermediate trades. Such a shortage will endogenously determine the allocation of intermediation rents among intermediaries, impacting the lenders' incentives on the margin, and as a result reducing the overall intermediation capacity of the system.

As a consequence of the cumulative nature of the moral hazard friction, we show that haircuts increase with the length of the intermediation chains: banks have to charge higher haircuts to induce the right behavior throughout the chain. Using a similar reasoning, we also show that equilibrium haircuts increase with the intensity of the moral hazard problem as well as the illiquidity of the collateral. As part of our analysis, we also study the role played by other important features of the collateral — beyond its liquidity — in determining the overall intermediation capacity of the financial system. We show that increasing the correlation between the quality of the collateral and the investment opportunities (e.g., by employing real-estate mortgage backed securities as collateral to fund other investments in the housing market) would reduce the intermediation capacity of the financial system as a whole. Intuitively, given that the borrower loses her collateral only in the state of the world in which her project fails, a higher correlation tightens the borrower's incentive compatibility constraint, thus forcing her corresponding lender to charge a higher haircut. Such overcollateralization, coupled with the cumulative nature of the moral hazard problem, would in turn reduce the intermediation capacity of the financial system, eventually leading to a potential market freeze. Yet another implication of our setting is that when this correlation is positive (negative), an increase in the riskiness of the collateral leads to higher (lower) haircuts, reduces (increases) the intermediation

capacity of the market, and increases (decreases) the possibility of credit freezes. The intuition for this result is similar: when positive, any increase in the riskiness of the securities employed as collateral shifts the proceeds from their liquidation from the bad to the good state of the world, thus increasing the incentives of any given borrower to invest in the inefficient project. Consequently, the corresponding lender ends up charging a higher haircut to discipline the borrower.

Finally, we investigate more general network structures. This serves two purposes. First, we can derive new insights on the intermediation paths that the capital will follow in the network in equilibrium. In fact, although moral hazard cumulates over an intermediation chain, the shortest path from the bank with excess capital to the one having the most profitable investment opportunity is not always the optimal one. This is because of the collateral constraints which may bind in equilibrium. In other words, if over the intermediaries along the shortest path experience a shortage of collateral, the lending bank would be forced to leave higher rents to the intermediaries in order to ensure that the capital is invested correctly. In turn, this alters his incentives to follow this shorter path in favor of longer paths where she will face higher liquidation costs, but lower dissipation of rents. Moreover, changing the collateral endowment to a bank on one particular intermediation path has effects on the other banks on the same path, but also on the rents captured by banks on other parallel paths, which shows the endogenous interconnections that arise when banks engage in secure lending transactions. Second, we show that more general structures, such as a core-periphery network, can be characterized very similarly to the characterization we provided for intermediation chains. The reason being that we can aggregate the collateral available to intermediaries on the same tier and build an equivalent chain for which our characterization still holds. This allows us to generalize our results to these network structures as well, and investigate how shocks to the network periphery might have a large impact on the entire intermediation network.

We remark that even though highly stylized, our setting captures several salient features of the repo market: borrowers can invest in projects with different risk characteristics; the collateral's value is uncertain and is potentially correlated with the investment's riskiness; and in case of a borrower's default, the liquidation of the collateral by the lender might be costly. More importantly, however, unlike most of the literature that mainly focuses on a single bilateral transaction, we study an inter-dealer market in which multiple financial institutions can borrow and lend to each other via bilateral repo contracts. Furthermore, in our setup all interest rates and haircuts are jointly determined in equilibrium.

Related Literature Our paper belongs to the recent literature that focuses on the emergence of intermediation chains in over-the-counter markets. For instance, [Glode and Opp \(2015\)](#) argue that, in the presence of asymmetric information, trading an asset through several heterogeneously informed intermediaries can reduce the extent of adverse selection between counterparties, and hence preserve the efficiency of trade. Other reasons have recently been proposed in the literature for the emergence of intermediation chains: heterogeneity in banks reserves ([Afonso and Lagos, 2014](#)), exposure to aggregate default risk ([Atkeson, Eisfeldt, and Weill, 2015](#)), search costs and heterogeneous

valuations (Hugonnier, Lester, and Weill, 2014; Shen, Wei, and Yan, 2015; Colliard and Demange, 2014). In contrast to these papers, we focus on the role of collateral in shaping the lending relationships among intermediaries on a financial network by affecting its intermediation capacity. The key friction in our model is the presence of moral hazard in the intermediation market. Our characterization results show that different features of the collateralizable assets — in particular their liquidity, volatility and distribution — are the key factors in determining the extent of intermediation in equilibrium.

The key role played by the repo market during the panic of 2007–8 has been highlighted by Gorton and Metrick (2012a,b), who document that haircuts rose dramatically at the onset of the crisis, mainly in response to increasing concerns about the liquidity of markets for bonds used as collateral. Based on these observations, they argue that the bilateral repo market suffered a run, which in turn amplified the financial crisis.¹ Relatedly, Krishnamurthy, Nagel, and Orlov (2014) examine data on the repo lending by money market funds and securities lenders and show that such liquidity providers became less willing to lend against risky or illiquid collateral. These studies highlight the importance of the collateral, and in particular its volume, riskiness and liquidity, in sustaining capital flows between financial intermediaries, all of which are central to our analysis.

On the theoretical side, a recent collection of papers, such as Parlatore Siritto (2015), Martin, Skeie, and von Thadden (2014), and Dang, Gorton, and Holmström (2013) study models of collateralized lending with applications to the repo market. Parlatore Siritto (2015) analyzes the trade-off between selling assets or pledging them as collateral and shows that collateralized debt contracts arise naturally to solve an asymmetric information problem about the borrowers' ability to repay.² Martin, Skeie, and von Thadden (2014) explore the role of liquidity and collateral constraints in determining the possibility of expectations-driven runs in the repo market. On the other hand, Dang, Gorton, and Holmström (2013) argue that overcollateralization arises to overcome the adverse selection problem that lenders may face in case of a borrower's default. They show that in such an environment, the arrival of public information about the quality of the collateral can lead to repo runs.³ We, in contrast, focus on a different friction from the aforementioned papers. In particular, we show that due to the cumulative nature of the moral hazard problem over chains of intermediaries, small shocks to the liquidity or availability of collateral assets can lead to large spikes in the haircuts, resulting in a potentially sharp collapse of the systems' intermediation capacity.

Our paper is also part of the recent but growing literature that focuses on the role of financial networks in shaping the fragility of the financial system. Initiated by the seminal works of Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000), this literature studies whether and how the ar-

¹On the contrary, Copeland, Martin, and Walker (2014) show that the tri-party repo market remained stable during the same period.

²Monnet and Narajabad (2012) provide conditions under which repurchase agreements can co-exist with asset sales. In particular, they show that, when borrowers are pairwise matched to lenders, repos become more prevalent as agents become more uncertain about the value of holding the asset.

³More recently, Lee (2015) shows that easier collateral circulation can potentially result in inefficient repo runs, whereas Eren (2014) provides a model in which the demand by dealer banks for funding liquidity determines repo haircuts and interest rates.

chitecture of the financial system can serve as a shock propagation and amplification mechanism. For example, [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2015\)](#) argue that the extent of contagion in a network of financial institutions linked via unsecured debt contracts is highly sensitive to the distribution of bilateral financial liabilities.⁴ We complement these studies by analyzing a model of secured lending in which interbank relationships are subject to moral hazard: rather than lending to the institutions with efficient investment opportunities, banks may divert their funds towards riskier, inefficient projects. We highlight a novel reason for the observed fragility of the secured lending market based on the distribution of the assets used as collateral over the financial network.

With a few recent exceptions, such as [Babus \(2014\)](#), [Farboodi \(2014\)](#) and [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2014\)](#), the literature on financial networks mainly studies intermediation while taking the interbank claims and liabilities as exogenous. In contrast, the terms of interbank contracts (such as interest rates and haircuts) as well as the realized paths of credit flow in our model are endogenously determined.

Finally, our work is also related to several recent papers that study the broad question of trade over networks, such as [Condorelli and Galeotti \(2012\)](#), [Choi, Galeotti, and Goyal \(2014\)](#), and [Manea \(2014\)](#). [Condorelli and Galeotti \(2012\)](#), for example, analyze a sequential model of trade of a single indivisible good over a general network, in which traders have incomplete information about one another's valuations. [Manea \(2014\)](#), on the other hand, studies a model in which the good is resold via successive bilateral bargaining between linked intermediaries and characterizes the endogenous structure of local monopolies and trading paths that arise in equilibrium.⁵ In contrast to these papers, our main focus is on the determinants of the financial system's intermediation capacity in the presence of moral hazard.⁶

The rest of the paper is organized as follows. Section 2 introduces the general model. Section 3 contains our main results where we show how the distribution, liquidity and riskiness of collateralizable assets determine the financial system's intermediation capacity. In Section 4, we generalize our results to a larger class of intermediation networks. Section 5 concludes. All proofs are presented in the appendix.

⁴Other related contributions include [Leitner \(2005\)](#), [Zawadowski \(2013\)](#), [Caballero and Simsek \(2013\)](#), [Alvarez and Barlevy \(2014\)](#), [Cabrales, Gottardi, and Vega-Redondo \(2014\)](#), [Glover and Richards-Shubik \(2014\)](#), [Elliott, Golub, and Jackson \(2014\)](#), [Glasserman and Young \(2015\)](#) and [Amini, Cont, and Minca \(forthcoming\)](#). For a survey of the earlier literature on financial networks, see [Allen and Babus \(2009\)](#). On the empirical side, [Di Maggio, Kermani, and Song \(2014\)](#) investigate the value of trading relationships and network centrality in the corporate bond market during the financial crisis, whereas [Gabrieli and Georg \(2014\)](#) study the liquidity allocation among European banks following the Lehman Brothers' insolvency and find that banks with higher centrality within the network had better access to liquidity and were able to charge larger intermediation spreads.

⁵Also related are [Goyal and Vega-Redondo \(2007\)](#), [Gofman \(2011, 2014\)](#), [Nava \(2015\)](#) and [Kotowski and Leister \(2014\)](#).

⁶The role played by the moral hazard friction in our model is reminiscent of [Kim and Shin \(2012\)](#), who use a multi-layered version of the contracting model of [Holmström and Tirole \(1997\)](#) to highlight that inter-firm credit, such as accounts receivable and payable, can solve recursive moral hazard problems that may arise in production chains. We, on the other hand, focus on collateralized lending relationships between financial institutions in a general network and show how the presence of moral hazard can lead to a potential credit freeze at the face of small shocks.

2 Model

Banks and Investments Consider an economy consisting of a collection of n risk-neutral financial institutions (henceforth, banks for short), which we denote by $N = \{1, \dots, n\}$. The economy lasts for three periods. At $t = 0$, bank i is endowed with A_i units of a bank-specific asset, which it can use as collateral for secured borrowing from other banks. If liquidated by its original owner at $t = 2$, each unit of the asset leads to one unit of proceeds, whereas banks other than the original owner can only recover a fraction $\alpha \leq 1$ of the value of the asset. This reduced-form parameter captures the idea that the collateral may be illiquid and hence, more costly for potential lenders to liquidate.⁷ The premature liquidation of the assets at $t = 1$ leads to a per-unit proceed of $\gamma < \alpha$, regardless of the identity of the liquidating bank, a parameter which we assume to be small.

At $t = 0$, each bank has access to a constant returns to scale investment opportunity (project), which has a $t = 1$ return of r_h with probability $1 - \phi_h$ and zero with probability ϕ_h . In addition to this investment opportunity, a single bank has access to a low-risk, low-return lumpy project of size k with rate of return $r_\ell < r_h$ and failure probability $\phi_\ell < \phi_h$. We assume that the projects' expected returns satisfy

$$r_\ell(1 - \phi_\ell) > r_h(1 - \phi_h), \quad (1)$$

i.e., it is always efficient to invest in the safer project. Without loss of generality, we index the bank with access to the efficient investment opportunity as bank 1.

Even though all banks have access to at least one investment opportunity, capital is scarce. In particular, at $t = 0$, a single bank — which we refer to as the *liquidity provider* — is endowed with k units of capital, whereas all other banks have no endowments of their own. The liquidity provider can either invest k in its own project(s), or lend it to other institutions. In view of inequality (1), if the liquidity provider is distinct from the bank with access to the efficient investment opportunity, it is always efficient for the former to (directly or indirectly) lend its capital to the latter.

Intermediation Network Even though there are potential gains from trade, the liquidity provider and the bank with access to the efficient project may not be able to trade with another directly, thus implying that other banks may need to act as intermediaries. The presence of such trade frictions may arise due to asymmetric costs of peer monitoring, adverse selection, absence of long-term interbank relationships, or pairwise commitment problems.⁸

Formally, we capture the presence of interbank trade frictions by an undirected, connected network $G = (V, E)$, where each vertex in V corresponds to a bank, and an edge $(i, j) \in E$ captures the

⁷In reality, various asset classes with a wide range of liquidities (ranging from the most liquid ones such as U.S. Treasuries to the least liquid ones such as mortgage-backed security or corporate bonds) are used as collateral.

⁸See Afonso, Kovner, and Schoar (2014) and Di Maggio (2014) for evidence supporting this assumption. Relatedly, Hugonnier, Lester, and Weill (2014) and Shen, Wei, and Yan (2015) show that several layers of intermediation naturally arise in a model of OTC market with search frictions and heterogeneous asset valuations.

possibility of trade between banks i and j .⁹ Note that the existence of an edge between two banks does not necessarily imply that they would enter into a lending agreement. Rather, as we describe next, interbank lending and borrowing decisions are determined endogenously.

Interbank Lending and Contracts Interbank lending occurs dynamically over $t = 0$ through short-term, secured lending (repo) contracts which have to be repaid at $t = 1$. At the beginning of period $t = 0$, each bank offers take-it-or-leave-it contracts to the set of banks it is connected to. More specifically, bank i offers a contract of the form (k_{ij}, R_{ij}, h_{ij}) to its potential counterparty j , where k_{ij} is the size of the loan, R_{ij} is the corresponding interest rate, and $h_{ij} \leq 1$ captures the extent of over-collateralization, which we refer to as the *haircut* on the loan.¹⁰ If bank i refuses to lend to bank j , it posts a contract with $k_{ij} = 0$.

After observing the set of contracts offered to it, each bank i can withdraw one or more of its own contract offers if it so wishes.¹¹ Given the set of remaining contracts, each bank j decides whether to accept or reject the offers it has received from its potential lenders. If bank j rejects i 's offer, bank i gets to keep the cash. If, on the other hand, bank j accepts the contract, it borrows k_{ij} from bank i and transfers $c_{ij} = k_{ij}/(1 - h_{ij})$ units of its illiquid asset to i as collateral.¹² At the last stage of the game at $t = 0$, all banks with cash on their hands decide to invest in the project(s) they have access to.

The repurchase leg of the repo agreements occurs once investment returns are realized, as each borrower bank j has to repurchase the collateral at price $R_{ij}k_{ij}$ from lender i if it can. If, on the other hand, the borrower is not capable of following through with the repurchase leg of the agreement, the lender has the right to keep the collateral.¹³

Finally, after the settlement of the repurchase agreements at $t = 1$, banks liquidate their holdings of the illiquid assets at $t = 2$ and consumption takes place.

Discussion of the Model The key underlying assumption of our model is that interbank interest rates and haircuts are neither contingent on the investment or lending decisions of the borrower

⁹A network is said to be *undirected* if $(j, i) \in E$ whenever $(i, j) \in E$. We say the network is *connected*, if there exists a path connecting every bank to every other bank in the network.

¹⁰The contracts in our model are similar to those of [Adrian and Shin \(2014\)](#). Also, to simplify the analysis, throughout the paper we assume that lenders have all the bargaining power. However, our qualitative results would be robust to allowing for a bargaining game in which the lender has the opportunity to make a take-it-or-leave-it offer with probability θ and the borrower has this opportunity with probability $1 - \theta$. See [Manea \(2014\)](#) for a discussion on the role of bargaining power in trade over networks.

¹¹This stage is introduced in order to allow a given bank i to make its lending decisions contingent on the contract posted by a potential creditor bank s . In other words, without the possibility of contract withdrawals, once bank i offers a contract to bank j , it already commits to lend to bank j on those terms regardless of the contract offered by bank s to i . Therefore, unless i can withdraw its contract, it may end up committing to lend a positive amount to bank j even if i 's lenders refuse to lend any money to it. See [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2014\)](#) for a similar argument in the context of unsecured lending contracts.

¹²We do not allow for rehypothecation of the collateral. We assume that the borrower prefers pledging the assets he owns as collateral rather than selling it, because it is costly to liquidate it.

¹³Note that for small enough values of γ , the borrower would default on its obligations to i , whenever it does not have enough cash available.

bank j , nor do they depend on the decisions of any other bank. This assumption implies that not only all pairwise interbank interactions are subject to moral hazard, but also that the agency problems are cumulative: as more banks act as intermediaries between the liquidity provider and the potential investors, the agency problems within the financial system intensify. The presence of such “cumulative” moral hazard problems lies at the core of our results.

We emphasize that it is fairly realistic to assume that pairwise interbank contracts have no or little contingencies on the potentially complex pattern of trades between all banks within the system, as information on such intricacies may not be available or contractible.

3 Intermediation Chains

In order to present the insights behind our model in the most transparent manner, we first focus on a simple environment in which the financial intermediation network is in the form of a *chain*. This allows us to obtain a parsimonious characterization of the equilibrium contracts and the financial system’s intermediation capacity.¹⁴ We study more general intermediation networks in Section 4.

We assume that each bank $i \in \{2, \dots, n - 1\}$ can only trade with two other banks labeled $i - 1$ and $i + 1$, whereas banks 1 and n can only trade with banks 2 and $n - 1$, respectively. Figure 1 depicts the corresponding financial network. We further assume that only bank 1 has access to the efficient investment opportunity (with rate of return r_ℓ and failure probability ϕ_ℓ), whereas bank n is the only bank with an excess liquidity of size k . Thus, the excess liquidity available to bank n can be invested in the efficient project only if banks 2 through $n - 1$ intermediate between banks 1 and n . As in our general model, we assume that all banks have access to the inefficient project with rate of return r_h and failure probability ϕ_h .



Figure 1. An intermediation chain: each bank i can only trade with banks $i - 1$ and $i + 1$, whereas banks 1 and n can only trade with banks 2 and $n - 1$, respectively. Bank n is the only bank with excess liquidity, whereas bank 1 is the sole bank with access to the efficient investment opportunity.

To characterize the equilibrium in this environment, denote the interest rate and collateral that bank $i + 1$ charges bank i by R_i and c_i , respectively. If the financial system manages to successfully intermediate the excess liquidity k available to bank n , then the payoff of bank $i + 1$ in the good state of the world in which the efficient project has a positive return is equal to $(R_i - R_{i+1})k + A_{i+1}$. On the other hand, if the efficient project fails (and for small enough values of γ), bank i has no cash available to repurchase the collateral at $t = 1$. Bank $i + 1$ thus gets to keep the collateral and obtains

¹⁴Such chains of intermediaries are common in several markets, such as interest rates swaps (Viswanathan and Wang, 2004), commodity markets (Weller, 2014), and the interbank market.

a payoff of αc_i at $t = 2$, whereas it loses c_{i+1} to its corresponding lender. Therefore, if intermediation is successful, the expected payoff of bank $i + 1$ is given by

$$\pi_{i+1} = (1 - \phi_\ell)(R_i k - R_{i+1} k + A_{i+1}) + \phi_\ell(\alpha c_i + A_{i+1} - c_{i+1}),$$

where recall that ϕ_ℓ is the failure probability of the efficient investment opportunity.

3.1 Market Fragility and Credit Freezes

To highlight how the juxtaposition of moral hazard and trade frictions can lead to market fragility and potential credit freezes, we first study an economy in which banks are not collateral-constrained, in the sense that they can post an arbitrarily large amount of the illiquid asset as collateral, if they choose to do so. Mathematically, this corresponds to the assumption that each bank's endowment of the collateralizable asset, A_i , is large enough.¹⁵ In the next subsection, we show how collateral scarcity can impact the extent of intermediation. We have the following result:

Proposition 1. *Suppose that banks are not collateral-constrained. Then, there exists n^* such that the interbank network efficiently intermediates the excess liquidity if and only if $n \leq n^*$, where n^* is increasing in α and satisfies $\lim_{\alpha \rightarrow 1} n^* = \infty$. Furthermore, the entire surplus goes to the liquidity provider bank n .*

Thus, any intermediation chain of length $n \leq n^*$ can be sustained in equilibrium, in the sense that the excess liquidity available to bank n is intermediated via banks 2 through $n - 1$ and is eventually invested in the efficient project by bank 1. More importantly, this is despite the fact that banks are restricted to writing simple repo contracts whose terms are not contingent on the lending or investment decisions of other institutions. Nevertheless, by charging a haircut, each lender bank i not only ensures that bank $i - 1$ does not divert the funds to the inefficient project, but also overcomes all moral hazard problems further down the chain. In other words, haircuts effectively “complete” the space of contracts.

The intuition behind this result is simple: by charging enough collateral, bank i can force its corresponding borrower, bank $i - 1$, to be more exposed to the downside risk of its investment decisions, hence making excessive risk-taking less appealing. In fact, bank i charges an interest rate and an haircut that simultaneously bind $i - 1$'s incentive compatibility and participation constraints, thus essentially eliminating the agency problem.

On the flip side, however, Proposition 1 also establishes that if the investment opportunity is too far from the bank with excess liquidity, that is $n > n^*$, then the interbank network is incapable of efficient intermediation: capital remains in the hands of bank n and is invested in the inefficient, riskier project. Note that even though bank i can always bring bank $i - 1$ all the way to its indifference point by charging a high enough haircut, the former can only recover a fraction $\alpha < 1$ of the value of the collateral. Thus, as the length of the chain (and hence, the cumulative severity of moral hazard

¹⁵At the end of 2006, the 10-Q filings of the major investment banks of the time, Goldman Sachs, Meryl Lynch, Lehman Brothers, Bear Sterns and J.P. Morgan, revealed that 47% of their assets were pledged as collateral in repurchase agreements.

problems) increases, the collateral that bank n needs to charge may be so large that it finds it optimal to simply invest in the inefficient project, as opposed to lending to $n - 1$.

The above argument also highlights that the maximum intermediation length that can be sustained in equilibrium crucially depends on the liquidity of the collateral, captured via parameter α . In fact as Proposition 1 shows, as the collateral becomes perfectly liquid, the equilibrium coincides with the first-best outcome regardless of the value of n . This result implies that when intermediaries employ highly liquid assets (such as Treasuries) as collateral, capital flows are efficient, even in the presence of agency problems. In contrast, if intermediaries' balance sheets are flooded by asset-backed securities and corporate bonds, efficient investment opportunities may be lost.

Finally, we remark that, in addition to the value of α , the maximum intermediation length also depends on the severity of the moral hazard problem. In particular, n^* is decreasing in the difference in returns of the inefficient and efficient projects, Δr . As the riskier investment becomes more attractive for the borrowing institutions, the agency problem between lenders and borrowers is exacerbated, shortening the intermediation chain that can be sustained in equilibrium.

Our next result provides an explicit characterization of the equilibrium interest rates and haircuts charged by the banks along the chain.

Proposition 2. *Suppose that banks are not collateral-constrained and that $n \leq n^*$. The equilibrium interest rate and haircut that bank $i + 1$ charges bank i are given by*

$$R_i = r_h \left(1 - \frac{\phi_h}{\zeta^{n^*-i}} \right) \quad (2)$$

and

$$h_i = 1 - \frac{\zeta^{n^*-i}}{r_h(1 - \phi_h)}, \quad (3)$$

respectively, where $\zeta = 1 + (1 - \alpha)\phi_\ell(1 - \phi_h)/\Delta\phi$.

Figure 2 depicts the equilibrium contracts along the chain. The key observation is that the equilibrium interest rates decrease as we move further away from the bank with the efficient investment opportunity, whereas the haircuts are increasing over the chain. Intuitively, the further away the excess liquidity is from the investment opportunity, the larger is the size of the cumulative moral hazard problem, thus requiring an increase in haircuts over the chain. On the other hand, given that each lender can discipline its corresponding borrower by charging either a high interest rate or a high haircut, it is natural that the two instruments function as substitutes. Thus, the interest rates are reduced as the haircuts are increased along the chain.

We have the following corollary to Proposition 2:

Corollary 1. *The equilibrium haircuts are decreasing in α and increasing in Δr , whereas the interest rates are increasing in α and decreasing in Δr .*

Thus, as the assets pledged as collateral become more illiquid (i.e., as α goes down), the haircuts increase while at the same time banks charge lower interest rates. This suggests that the emergence

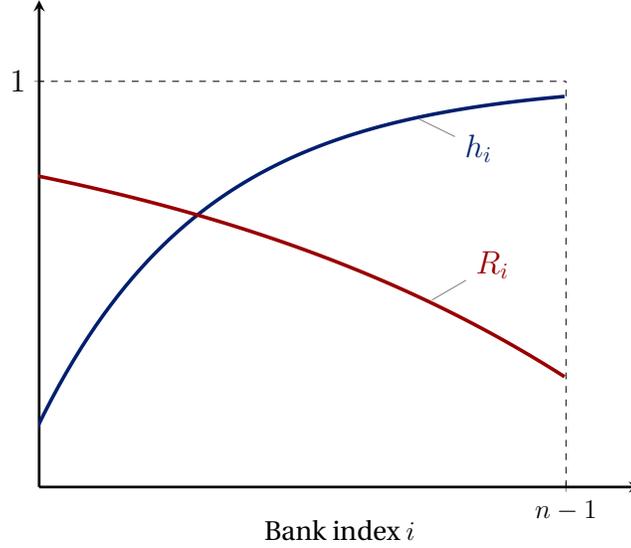


Figure 2. The equilibrium repo contract (k, R_i, h_i) offered by bank $i + 1$ to bank i in the absence of collateral constraints. The red and blue lines, respectively, depict the interest rate R_i and haircut h_i as a function of i . As long as $n \leq n^*$, all excess liquidity k is eventually invested in the efficient project.

of haircut in our model not only depends on the severity of the agency problems, but also on the presence of frictions in the secondary market for the collateral. As expected, each lender is willing to lend less against a less liquid collateral. Furthermore, Corollary 1 shows that increasing Δr also leads to higher haircuts across the board. Note that a higher Δr increases the borrowers' incentives to deviate and invest in the inefficient project, which in equilibrium makes it optimal for the lender to charge a higher haircut. Such an increase in haircuts in turn implies that the interest rates are optimally set to lower levels.

A key implication of our results is that small shocks can propagate throughout the chain, with possibly significant implications for the terms of equilibrium contracts and hence, the efficiency of the outcome. In particular, a slight decrease, say, in the return of the efficient project available to bank 1 not only affects the investment incentives of that bank, but also intensifies the agency problems between any pair of banks over the chain. More importantly, since the moral hazard problems are cumulative, such a small shock may lead to large spikes in equilibrium haircuts charged by banks further down the chain. In fact, given that n^* is decreasing in Δr , a small shock to bank 1's investment opportunity may in fact lead to a complete *credit freeze*, in the sense that bank n refrains from lending its excess liquidity altogether, and instead invests in the inefficient project.

Similarly, we can also investigate the role of idiosyncratic shocks in shaping the transmission of risk and the build up of fragility throughout the system. For instance, consider the case in which the collateral of a single bank i is less liquid, i.e. $\alpha_i = \underline{\alpha} < \alpha$. This captures the possibility that a single intermediary pledges a less liquid asset (say, mortgage-backed securities, as opposed to triple-A assets) as collateral. We can employ our recursive characterization to solve for the haircuts and the interest rates that would emerge in equilibrium. Figure 3 visualizes the effect of this shock on the haircuts and yields for all market participants. Interestingly, this observation suggests that a change

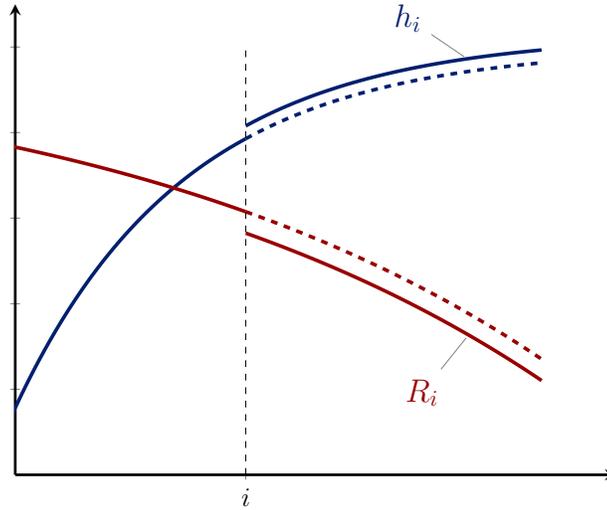


Figure 3. A negative shock to the liquidity of the collateralizable asset available to bank i leads to higher haircuts throughout the chain.

in the collateral liquidity of a single bank can adversely affect the terms of trades for the whole market. In other words, due to the intertwined nature of the links between institutions, employing lower quality collateral by a peripheral dealer would have far-reaching effects, as even dealers employing collateral of higher quality may face larger haircuts.

3.2 Intermediation Capacity

Our analysis in the previous subsection shows how the cumulative nature of moral hazard problems over the network of interbank relationships may lead to fragility in intermediation. In this subsection, we show that the extent of this fragility crucially depends on the distribution of endowment of collateralizable assets among the intermediaries. To this end, we relax the assumption that banks have infinite reserves of collateralizable assets, and instead consider the more realistic scenario in which there is an upper limit A_i to the value of assets that bank i can pledge as collateral, implying that the intermediaries' collateral constraint may bind in equilibrium. The presence of such a limit in conjunction with the cumulative moral hazard problem means that there is a maximum amount of liquidity that can be intermediated by the system. More specifically, we define:

Definition 1. The *intermediation capacity* of the financial system, k_{\max} , is the maximum amount of capital that the liquidity provider is willing to eventually invest in the efficient project.

The intermediation capacity arises due to the fact that bank n faces a trade-off in how to allocate its excess liquidity between the efficient project (which requires intermediation) and the riskier investment opportunity. On the one hand, it has an incentive to invest in the more efficient project by pushing more of its capital through the chain of intermediaries. On the other hand, a large enough investment in the efficient project would imply that the collateral constraint of at least one intermediary would bind in equilibrium, which means that the participation constraint of such intermediaries cannot bind. Consequently, to overcome the agency problems over the chain, some of the

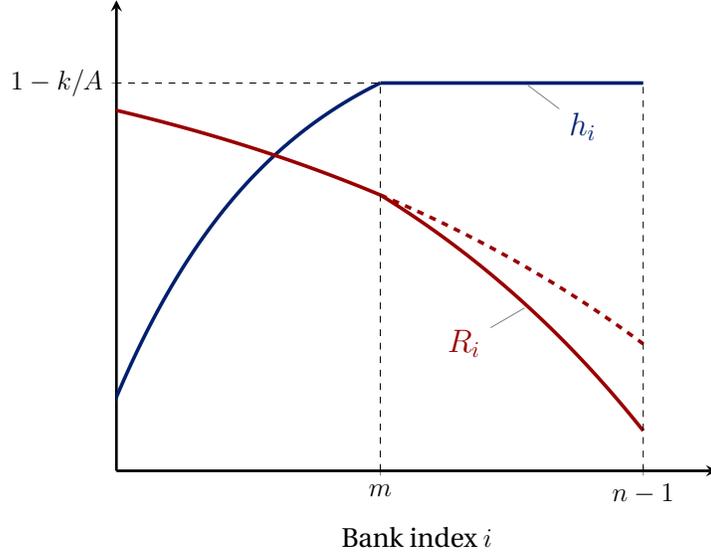


Figure 4. The equilibrium contract (k, R_i, h_i) charged by bank $i + 1$ to bank i , in the presence of collateral constraints. The red and blue lines, respectively, depict the interest rate R_i and haircut h_i as a function of i . Bank m is the first bank whose collateral constraint binds. The dashed red line depicts the interest rates had the collateral constraints not been binding.

surplus has to be shared as intermediation rents along the chain, thus reducing the attractiveness of investing in the efficient project.

Figure 4 depicts this trade-off for an economy in which all banks have an identical endowment of the collateralizable asset, i.e., $A_i = A$ for all i . Recall from Proposition 2 and Figure 2 that, for intermediation to be successful, equilibrium haircuts increase monotonically over the intermediation chain. However, given the limited amount of collateralizable assets available to the banks, the haircuts cannot surpass $h_{\max} = 1 - k/A$. Thus, in order to make the capital flow possible, each lender bank $i > m$ needs to leave some rents to its corresponding borrower bank $i - 1$ by cutting the interest rate R_{i-1} above and beyond what it would have been in the absence of collateral constraints. Eventually, the cumulative size of these rents would become so large that bank n finds it optimal to reduce the investment in the efficient project and instead, divert some of its excess liquidity to the less efficient project.

Our next result formalizes the above argument and characterizes the intermediation capacity of the financial system for a general distribution of collateralizable assets.

Proposition 3. *Suppose that $n \leq n^*$. The intermediation capacity of the chain is*

$$k_{\max} = \begin{cases} \min_{i \leq m} \frac{A_i \zeta^{n^* - i}}{r_h (1 - \phi_h)} & \text{if } m \geq 1 \\ \infty & \text{otherwise} \end{cases}, \quad (4)$$

where

$$m = \left\lceil \frac{n \log \omega - n^* \log \zeta}{\log \omega - \log \zeta} \right\rceil - 1 \quad (5)$$

is the first bank on the chain whose collateral constraint binds and $\omega = (1 - \phi_\ell)/\Delta\phi$.

The above result highlights the importance of collateralizable assets in the financial institutions' role as intermediaries: the shortage of such assets reduces the ability of lenders to discipline potential deviations by the borrowers. In fact, as predicted by Proposition 3, the intermediation capacity of the financial system is increasing in the banks' endowments A_i .

Proposition 3 has two main implications. First, it highlights that financial system's intermediation capacity crucially depends on how the endowments of collateralizable assets A_i are distributed throughout the system, as opposed to only the aggregate amount in the economy (given by $A_1 + \dots + A_n$). Put differently, in the presence of trade frictions (captured in our model by the means of the intermediation network), the aggregate amount of collateral is not a sufficient statistics for the financial system's intermediation capacity. This is a consequence of the fact that if one of the intermediaries is not able to post enough collateral and as a result demands higher rents, its lender will have an incentive to divert funds towards the riskier investment opportunity, hence, reducing the system's intermediation capacity. Furthermore, given that welfare is proportional to the system's intermediation capacity, the above result establishes that the *distribution* of collateral has a first-order impact on how efficiently banks can intermediate trade.

Second, Proposition 3 also highlights how each bank's endowment of collateralizable assets determines the intermediation capacity of the network. In particular, equation (4) shows that only the endowments of the m banks closer to the investment opportunity matter for the value of k_{\max} . Consequently, all else equal, reallocating collateralizable assets from banks closer to the liquidity provider to the ones closer to the investment opportunity would increase efficiency. Notice, however, that this is not to say that the collateral holdings of other banks do not matter for banks' profits. Rather, it is only the amount of assets in the hands of the intermediaries closer to the investment opportunity that ends up shaping the network's intermediation capacity.

As a final remark, we reemphasize that even though the financial system can efficiently intermediate up to k_{\max} units of capital, not all the surplus would go to the liquidity provider bank n . Rather, banks $\{m + 1, \dots, n - 1\}$ may also get a portion of the surplus as intermediation rents. In fact, it is only because of such rents that the system cannot efficiently intermediate beyond k_{\max} . Furthermore, note that not only the intermediation capacity, but also the allocation of intermediation rents depends on the distribution of collateralizable assets throughout the financial system. This observation highlights that, unlike Farboodi (2014), the allocation of surplus between different parties are determined endogenously in our model.

We have the following corollary to Proposition 3.

Corollary 2. *Suppose that $n \leq n^*$. The intermediation capacity of the chain, k_{\max} , is decreasing in Δr and n , and increasing in α .*

Intuitively, increasing Δr increases the borrowers' incentives to deviate and invest in the riskier project, hence intensifying the agency problem between any pair of banks. Similarly, the larger the distance between the liquidity provider and the eventual borrower is, the more severe the cumulative

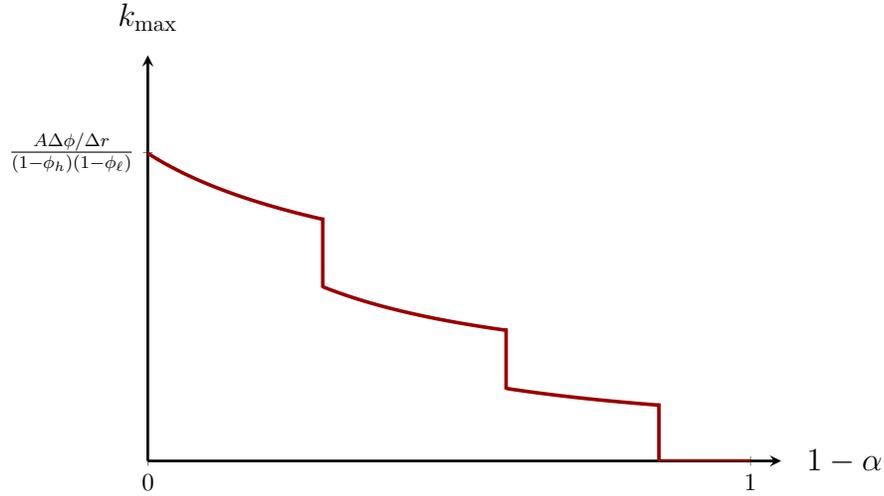


Figure 5. Intermediation capacity k_{\max} as a function of the collateral's illiquidity, measured in terms of $1 - \alpha$. The intermediation capacity decreases as the collateral becomes more illiquid.

moral hazard problem within the chain becomes. Thus, as the above result shows, increasing Δr and n would reduce the intermediation capacity of the system.

Corollary 2 also shows that intermediation capacity is higher the more liquid the collateral asset is, as each bank is willing to lend more against such assets. Figure 5 depicts k_{\max} as a function of $1 - \alpha$ in an economy in which all banks have an identical endowment A of the collateralizable asset. The key observation is that not only the intermediation capacity of the system decreases as the collateral becomes more illiquid, but also that it changes discontinuously as a function of $1 - \alpha$. Therefore, a slight change in the asset's liquidity may result in significant drops in the system's intermediation capacity. Furthermore, a sufficient reduction in α may lead to a complete credit freeze, whereby the intermediation capacity of the system collapses to zero.

The emergence of discontinuities in the system's intermediation capacity is due to the fact that for all banks $i \geq m$, the corresponding lender bank $i + 1$ can overcome the moral hazard problem only if it leaves some rents with i . Therefore, as the collateral becomes less liquid, at some point the collateral constraint of bank $m - 1$ would also start to bind, reducing the marginal benefit of lending one more unit of capital to bank $m - 1$ (as now a fixed fraction of the surplus has to be left with $m - 1$). This discontinuous change in the marginal benefit of intermediation leads to the a discontinuous drop in the system's overall intermediation capacity, which is another reason why the intermediation network becomes fragile as small changes to the liquidity of the collateral lead to large changes in the amount of capital that banks are able to intermediate. Finally, recall from Proposition 1 that the maximum length of the intermediation chain that can be sustained in equilibrium, i.e., n^* , is increasing in α . Thus, a sufficient decrease in the collateral asset's liquidity may bring n^* below n , thus leading to a complete credit freeze.

3.3 Risky Collateral

Our results so far depended on the assumption that each unit of the illiquid asset used as collateral in the secured lending contracts has a fixed, deterministic value. In reality, however, an important concern for the lenders is the riskiness of the securities pledged to them as collateral. In this section, we show how the presence of such risks affects the equilibrium and the intermediation capacity of the system. More specifically, we assume that the liquidation value of the collateral not only depends on the timing of liquidation and the identity of the liquidating bank, but also on the aggregate state of the world.

To capture this idea formally, suppose that the liquidation value of the collateral by its original owner at $t = 2$ is a random variable z , with standard deviation σ and expected value normalized to 1. As before, we assume that any other bank can only recover a fraction $\alpha \leq 1$ of the value of the asset. Furthermore, let ρ denote the correlation between z and the aggregate state of the world, defined as the success or failure of the inefficient project. Throughout, we assume that the returns of the efficient and inefficient projects are independent.

This extension of the basic setup aims to capture two important features of the repo arrangements between financial institutions. First, the riskiness of the collateral captures the lenders' necessity to protect themselves from the risk of holding assets of lower than expected quality. Second, the correlation ρ might capture several possibilities. For instance, the borrowing institution may invest in the housing market by lending directly to risky households or purchasing collateralized debt obligations (CDO), while at the same time pledging mortgage-backed securities already held in its portfolio as collateral in repo transactions. In this way, the lending institution may end up significantly more exposed to the housing market through both the investment of the borrower — which would increase its counterparty risk — as well as the collateral, whose proceeds would decrease in the state of the world in which the borrower defaults. In such a scenario, a negative shock to the housing market would be even more problematic for the lender as the collateral will turn out to be less valuable. These concerns seem to have been particularly important at the onset of the financial crisis, with liquidity providers pulling out from their lending relationships due to an increase in the perceived risk of the collateral, for example for the cases of ABCP and ABS, and due to an increase in their exposure to a declining housing market.

Proposition 4. *Suppose that $\rho\sigma \leq \left(\frac{\Delta\phi}{1-\phi_\ell}\right) / \sqrt{\phi_h(1-\phi_h)}$.*

- (a) *If banks are not collateral-constrained, then there exists n^* decreasing in $\rho\sigma$, such that the interbank network efficiently intermediates the excess liquidity if and only if $n \leq n^*$.*
- (b) *All interbank interest rates are decreasing in $\rho\sigma$, whereas haircuts increase with $\rho\sigma$.*
- (c) *Suppose that $n \leq n^*$ and that each bank is endowed with A units of the illiquid asset. Then, the intermediation capacity of the market is decreasing in $\rho\sigma$.*

The above result thus generalizes Propositions 1–3 to the case in which the returns of the securities used as collateral are potentially risky.¹⁶ It shows that when banks are not collateral-constrained, an increase in the correlation between the collateral’s liquidation value and the inefficient project’s returns not only increases the equilibrium haircuts, but may also lead to a complete credit freeze. In particular, increasing ρ may reduce n^* to such an extent that the liquidity provider would refrain from lending altogether. Similarly, when banks are constrained by how much collateral they can put up, increasing the correlation between the returns of the securities and the projects would reduce the intermediation capacity of the financial system as a whole.

The intuition underlying this result is simple: given that the borrower loses her collateral only in the bad state of the world, increasing ρ tightens its incentive compatibility constraint, thus forcing its corresponding lender to charge a higher haircut. Furthermore, due to the cumulative nature of the moral hazard problem in our model, a higher ρ also reduces the intermediation capacity of the system and may eventually lead to a credit freeze. Note that this result is in contrast to that of [Parlatore Siritto \(2015\)](#), who shows that keeping the (unconditional) expected return of the asset fixed, an increase in the correlation between the success of the investment made by the banks and the future dividends paid by the asset increases the asset’s debt capacity.

Yet another implication of Proposition 4 is that when ρ is positive (negative), an increase in the riskiness of the collateral leads to higher (lower) haircuts, increases (decreases) the possibility of market freezes, and reduces (increases) the intermediation capacity of the market. The intuition for this result is identical: when $\rho > 0$, any increase in the riskiness of the securities, i.e., a higher σ , shifts the proceeds from liquidating the collateral from the bad to the good state of the world, thus increasing the incentives of any given borrower to invest in the inefficient project. Consequently, the lender ends up charging a higher haircut to discipline the borrower.

4 Intermediation Networks

Having established that the shortage of collateral has a first order impact on the capacity of the financial system to efficiently allocate capital among banks, we now show that the collateral distribution has other non-trivial implications in more complex network structures. Specifically, in this section we focus on a more general class of intermediation networks, such as core-periphery networks, and show that the insights highlighted in the previous section carry over. More importantly, such an analysis enables us to investigate how the distribution of collateralizable assets affects another important dimension of the interbank lending market, namely, the *intermediation path(s)* over which capital flows from liquidity providers to banks with access to investment opportunities and the distribution of rents among intermediaries.

From our earlier discussions, one may expect that the cumulative nature of moral hazard frictions in our model would suggest that capital should always follow the shortest possible path from the cash lenders to the the investing banks. For instance, consider the network depicted in Figure

¹⁶We provide the closed-form expression of the intermediation capacity for the system in the Appendix.

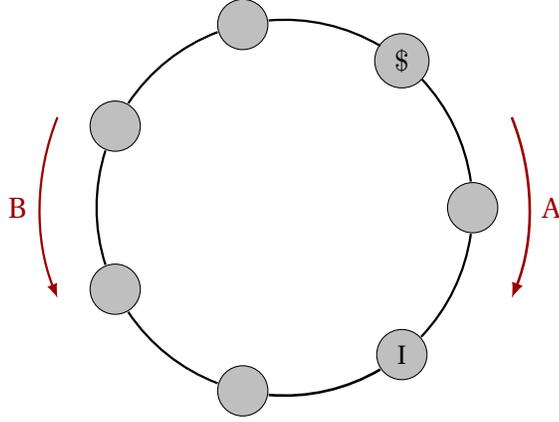


Figure 6. An intermediation network with two parallel intermediation paths.

6, where the excess cash in the market can potentially flow from the liquidity provider, denoted by “\$”, through a single intermediary to the investing bank denoted by “I” (path A) or alternatively, via a path in which there are multiple banks intermediating the transactions (path B). Since each bilateral transaction is subject to moral hazard frictions, the cash lender should find it profitable to employ only the shortest path whenever $\alpha < 1$. However, this intuition is incomplete. In fact, due to the presence of potentially binding collateral constraints, the liquidity provider faces a trade-off similar to the one highlighted in Section 3: even though the shortest path minimizes the cumulative effect of moral hazard and the corresponding costs generated by the collateral’s illiquidity, lenders may be forced to leave some of the surplus as rents with the intermediaries whose collateral constraints bind. This, in turn, alters the liquidity provider’s incentives to follow shorter paths in favor of routing a fraction of its excess capital through longer paths over which the banks’ collateral constraints may not yet be binding. Thus, depending on the size of excess cash, k , the liquidity provider may find it optimal to lend to multiple counterparties, even if they are not part of the shortest intermediation path to the investment opportunity. Nevertheless, we remark that, *ceteris paribus*, the volume of capital intermediated is always decreasing in the length of the path connecting the liquidity provider to the investment opportunities.

The above discussion naturally leads to the following question: how are intermediation rents affected when banks’ endowments of collateralizable assets change? We have the following result:

Proposition 5. *Consider an intermediation network with multiple non-intersecting parallel paths connecting the liquidity provider to the efficient investment opportunity. Increasing the endowment of collateralizable assets of a bank weakly increases (decreases) the intermediation rent of other banks on that path (on other paths).*

Intuitively, as the endowments of collateralizable assets of banks on a given path are increased, the endogenous flow of capital over that path would increase, reducing the rents that can get captured by banks located elsewhere in the network. The above result thus shows that changes to the endowment of collateralizable assets of a single bank can have non-trivial impacts on the allocation

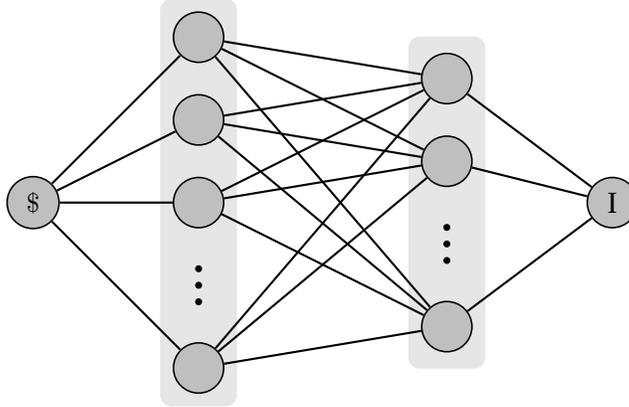


Figure 7. A multi-tiered intermediation network

of intermediation rents for *all* banks in the networks, which in turn affects the overall intermediation capacity of the network, as well as the path that capital follows in equilibrium.

We now turn to the characterization of the intermediation capacity of a more general class of intermediation networks, known as *tiered intermediation networks*. Formally, each bank in such a network belongs to one of multiple tiers (say, tier i) and is connected to all banks belonging to tiers $i + 1$ and $i - 1$. Figure 7 depicts a tiered intermediation network consisting of four bank tiers. It is immediate to see that the intermediation chain studied in Section 3 is a simple tiered network in which each tier consists of a single bank. We have the following result:

Proposition 6. *The intermediation capacity of a tiered intermediation network with s_i banks on the i -th tier each of which endowed with A units of collateralizable asset is equal to the intermediation capacity of an intermediation chain in which the i -th bank is endowed with $s_i A$ units of the collateralizable asset.*

Proposition 6 thus generalizes the characterization we provided for intermediation chains to the class of tiered financial networks. More specifically, it shows that essentially one can aggregate the endowments of collateralizable assets available to intermediaries belonging to the same tier and build an equivalent chain for which our characterization still holds.

Beyond generalizing our earlier results on intermediation chains, Proposition 6 can also be used to characterize the intermediation capacity of the so-called *core-periphery* networks, in which a group of peripheral institutions are linked to a set of core intermediaries that are fully connected to one another. Such structures are of particular interest due to the fact they are observed in a number of different over the counter markets (Craig and von Peter, 2014; Li and Schürhoff, 2014; Afonso et al., 2014). Proposition 6 implies that our results from Section 3 carry over to this more general class of networks. For instance, if a peripheral bank pledges a collateral of lower quality (which is, say, less liquid or more volatile), the consequences would ripple across the whole network, manifesting themselves as large swings in haircuts such as those observed at the onset of the recent crisis.

5 Conclusions

In this paper, we study a dynamic model of financial intermediation in which interbank lending is subject to moral hazard and show that in the presence of such agency problems, small shocks can lead to large spikes in the haircuts and a sudden collapse in the intermediation capacity of the financial system. In particular, we argue that due to the cumulative nature of the moral hazard problems over chains of intermediaries, a slight change in the perception about the collateral's liquidity or the expected returns of different projects may lead to a complete credit freeze. We also show how the quality of collateral — in particular its liquidity, volatility and availability — as well as its allocation throughout the financial system plays a central role in determining the system's capacity to allocate funds efficiently. Specifically, the distribution of collateral among banks plays a key role in shaping the intermediation capacity of the network and its potential fragility. We show that to maximize the intermediation capacity of the network, collateral need to be allocated to the intermediaries close to the investment opportunity.

Appendix

A Proofs

Proof of Proposition 1

Note that in any equilibrium of the game, no bank other than n would invest in the inefficient project. This is due to the fact that in any candidate equilibrium in which some other bank invests in the inefficient project, bank n can deviate by investing in the inefficient project directly and obtaining a strictly higher payoff. Thus, there are only two possible lending patterns that are consistent with equilibrium: either (i) bank n invests directly in the inefficient project; or (ii) for all $i \in \{2, \dots, n\}$, bank i lends to bank $i - 1$ and bank 1 eventually invests in the efficient asset. Note that in the latter equilibrium, the expected profit of bank $i + 1$ is equal to

$$\pi_{i+1} = (1 - \phi_\ell)(R_i - R_{i+1})k + \phi_\ell(\alpha c_i - c_{i+1}) + A, \quad (6)$$

where R_i and c_i are the interest rates and collateral that $i + 1$ charges i , with the convention that $R_0 = r_\ell$ and $R_n = c_0 = c_n = 0$. The first term on the right-hand side is the payoff of the bank in the good state of the world. On the other hand, in the bad state of the world, the bank obtains αc_i from liquidating the collateral posted by i , while at the same time losing c_{i+1} , as bank $i + 2$ would not return its collateral.¹⁷

We now derive conditions under which full lending over the chain can be sustained as an equilibrium. Consider a candidate equilibrium in which bank $i + 1$ obtains the excess cash. As already mentioned above, this is consistent with equilibrium if and only if the contract (R_{i+1}, c_{i+1}) offered by bank $i + 2$ induces $i + 1$ to subsequently lend the liquidity to bank i — as opposed to investing it in the inefficient project. Thus, bank $i + 1$ chooses the contract (R_i, c_i) offered to i that solves the following problem:

$$\begin{aligned} \max_{R_i, c_i} \quad & \pi_{i+1} \\ \text{s.t.} \quad & \pi_i \geq (1 - \phi_h)(r_h - R_i)k - \phi_h c_i + A \\ & \pi_i \geq A. \end{aligned} \quad (7)$$

Denote the Lagrange multipliers corresponding to the incentive compatibility and participation constraints in the above problem with λ_i and μ_i , respectively. The first-order conditions imply

$$\begin{aligned} \mu_i &= 1 - \phi_\ell + \alpha \phi_\ell \\ \lambda_i &= \phi_\ell(1 - \phi_\ell)(1 - \alpha)/\Delta\phi, \end{aligned}$$

guaranteeing that both constraints bind in the optimal solution. The fact that the participation constraints of all banks bind implies that $\pi_i = A$ for $i \neq n$. Hence, summing equation (6) over i leads

¹⁷Note that as long as $c_i \leq R_i k / \gamma$, bank i chooses to default on its commitment to $i + 1$ as opposed to repurchasing the collateral. Hence, for small enough γ , all banks default on their obligations in the bad state of the world.

to

$$\pi_n = (1 - \phi_\ell)r_\ell k - \phi_\ell(1 - \alpha) \sum_{i=1}^{n-1} c_i + A. \quad (8)$$

Thus, to determine bank n 's payoff when it lends to bank $n - 1$, it is sufficient to determine the size of the collateral demanded by each lender bank from its respective borrower. On the other hand, the fact that both constraints in the above problem bind, it is immediate that the pair (R_i, c_i) is the solution to the system of equations

$$(1 - \phi_\ell)(R_{i-1} - R_i)k + \phi_\ell(\alpha c_{i-1} - c_i) = 0 \quad (9)$$

$$(1 - \phi_h)(r_h - R_i)k - \phi_h c_i = 0. \quad (10)$$

Eliminating the interest rates from the above equations implies that the optimal collateral values are given by the following recursion

$$c_i = \zeta c_{i-1}, \quad (11)$$

with the initial condition

$$c_1 = (1 - \phi_h)(1 - \phi_\ell)\Delta r k / \Delta \phi, \quad (12)$$

where

$$\zeta = 1 + \frac{(1 - \alpha)\phi_\ell(1 - \phi_h)}{\Delta \phi}. \quad (13)$$

Substituting the above in (8) implies that the expected profit of bank n is equal to

$$\pi_n = (1 - \phi_\ell)r_\ell k - (1 - \phi_\ell)(\zeta^{n-1} - 1)\Delta r k + A.$$

Thus, regardless of the value of k , bank n prefers to lend to bank $n - 1$ if and only if

$$(1 - \phi_\ell)r_\ell - (1 - \phi_\ell)(\zeta^{n-1} - 1)\Delta r \geq (1 - \phi_h)r_h$$

which holds as long as $n \leq n^*$, where n^* satisfies¹⁸

$$\zeta^{n^*-1} = \frac{r_h \Delta \phi}{(1 - \phi_\ell)\Delta r}. \quad (14)$$

Given that $\lim_{\alpha \rightarrow 1} \zeta = 1$, it is immediate that n^* becomes arbitrarily large as α converges to 1. \square

¹⁸To simplify derivations, we assume that the solution to this equation is an integer. All our results and their economic insights would remain valid if the solution is not an integer.

Proof of Proposition 2

From (11) in the proof of Proposition 1, we already know that $c_i = \zeta c_{i-1}$, which implies that $c_i = \zeta^{i-1} c_1$. This observation along with (12) imply that the size of the collateral posted by bank i with bank $i - 1$ is equal to

$$c_i = \zeta^{i-1} (1 - \phi_h)(1 - \phi_\ell) \Delta r k / \Delta \phi.$$

Using the fact that $h_i = 1 - k/c_i$ and the definition of n^* in (14) leads to (3).

To derive the equilibrium interest rates, recall from the proof of Proposition 1 that the incentive compatibility and participation constraints both bind at the optimal solution, implying that the equilibrium interest rate and collateral that bank $i + 1$ charges bank i satisfy the system of equations (9) and (10). Eliminating c_i and solving for R_i imply that the interest rates are determined via the recursion

$$\begin{aligned} R_i &= \zeta R_{i-1} - (1 - \alpha)(1 - \phi_h) \phi_\ell r_h / \Delta \phi \\ &= \zeta R_{i-1} - (\zeta - 1) r_h, \end{aligned}$$

for $i \geq 2$ with the initial condition

$$R_1 = r_h - \phi_h(1 - \phi_\ell) \Delta r / \Delta \phi.$$

Solving for the above recursion implies that $R_i = r_h - \zeta^{i-1} (r_h - R_1)$ and hence,

$$R_i = r_h - \left(\frac{\phi_h(1 - \phi_\ell) \Delta r}{\Delta \phi} \right) \zeta^{i-1},$$

which coincides with (2) once one replaces for ζ^{n^*-1} from (14). □

Proof of Corollary 1

Recall from the proof of Proposition 2 that the interest rate charged by bank $i + 1$ is given by

$$R_i = r_h - \left(\frac{\phi_h(1 - \phi_\ell) \Delta r}{\Delta \phi} \right) \zeta^{i-1}.$$

Given that ζ is decreasing in α it is immediate that R_i is increasing in α . To obtain the comparative statics with respect to Δr , note that increasing r_ℓ increases R_i . On the other hand,

$$\frac{\partial R_i}{\partial r_h} = 1 - \frac{\phi_h(1 - \phi_\ell)}{\Delta \phi} \zeta^{i-1}.$$

Given that both ζ and $\phi_h(1 - \phi_\ell)/\Delta \phi$ are greater than 1 it is immediate that the right-hand side of the above equality is negative, implying that R_i is decreasing in r_h . Consequently, R_i is decreasing in Δr .

As for the haircuts, recall from the proof of Proposition 2 that the collateral demanded by bank $i + 1$ is given by

$$c_i = \zeta^{i-1} (1 - \phi_h)(1 - \phi_\ell) \Delta r k / \Delta \phi.$$

Given that ζ is decreasing in α , it is immediate that c_i is also decreasing in α . The above equality also implies that c_i is increasing in Δr , completing the proof. □

Proof of Proposition 3

Recall from the proof of Proposition 1 that bank $i+1$ chooses the interest rate and collateral it charges bank i as solutions to the following problem:

$$\max_{R_i, c_i} \quad \pi_{i+1} \quad (15)$$

$$\text{s.t.} \quad \pi_i \geq (1 - \phi_h)(r_h - R_i)k - \phi_h c_i + A_i \quad (16)$$

$$\pi_i \geq A_i \quad (17)$$

$$c_i \leq A_i, \quad (18)$$

where $\pi_i = (1 - \phi_\ell)(R_{i-1} - R_i)k + \phi_\ell(\alpha c_{i-1} - c_i) + A_i$ is the expected profit of bank i when the excess cash is invested in the efficient project and inequality (18) captures the fact that the collateral bank $i+1$ charges bank i cannot exceed i 's endowment of the collateralizable asset.

Denote the Lagrange multipliers corresponding to the incentive compatibility, participation and collateral constraints with λ_i , μ_i and η_i , respectively. The corresponding first-order conditions are thus given by

$$\lambda_i = (1 - \mu_i)(1 - \phi_\ell)/\Delta\phi$$

$$\eta_i = (1 - \mu_i)(1 - \phi_\ell) + (\alpha - \mu_i)\phi_\ell.$$

It is immediate that the incentive compatibility constraint (16) binds in equilibrium, as the above system of equations has no non-negative solution with $\lambda_i = 0$. Therefore, the interest rate charged by bank $i+1$ is determined recursively as

$$R_i k = \left(\frac{1 - \phi_\ell}{\Delta\phi} \right) R_{i-1} k + c_i + \frac{1}{\Delta\phi} \left(\alpha \phi_\ell c_{i-1} - (1 - \phi_h) r_h k \right),$$

with the initial condition $R_1 k = c_1 + (1 - \phi_\ell) r_\ell k / \Delta\phi - (1 - \phi_h) r_h k / \Delta\phi$. Solving the above recursion implies that the interest rate that bank n charges bank $n-1$ is given by

$$R_{n-1} k = \left(\frac{1 - \phi_\ell + \alpha \phi_\ell}{\Delta\phi} \right) \sum_{j=1}^{n-2} c_j \left(\frac{1 - \phi_\ell}{\Delta\phi} \right)^{n-j-2} - \left(\frac{1 - \phi_\ell}{\Delta\phi} \right)^{n-1} \Delta r k + c_{n-1} + r_h k. \quad (19)$$

On the other hand, recall that bank n finds intermediation through the chain profitable if and only if $\pi_n \geq (1 - \phi_h) r_h k + A_n$, where $\pi_n = (1 - \phi_\ell) R_{n-1} k + \alpha \phi_\ell c_{n-1} + A_n$ is the expected profit of bank n . Replacing for R_{n-1} from (19) thus implies that bank n decides to lend to bank $n-1$ if and only if

$$\left(\frac{1 - \phi_\ell + \alpha \phi_\ell}{\Delta\phi} \right) \sum_{j=1}^{n-1} \omega^{n-j-1} c_j + r_h k - \omega^n \Delta r k \geq 0, \quad (20)$$

where $\omega = (1 - \phi_\ell)/\Delta\phi$. Therefore, the intermediation capacity of the financial system is given by

$$k_{\max} = \arg \max_k \left\{ \left(\frac{1 - \phi_\ell + \alpha \phi_\ell}{\Delta\phi} \right) \sum_{j=1}^{n-1} \omega^{n-j-1} c_j + r_h k - \omega^n \Delta r k \right\}. \quad (21)$$

It is thus sufficient to determine the equilibrium collaterals demanded by the lenders in the chain. To this end, recall that the incentive compatibility constraint (16) always binds. Consequently, only one of the participation or the collateral constraints (respectively, (17) and (18)) would be tight. Assuming that the collateral constraint does not bind, solving for c_i from (16) and (17) implies that $c_i = \zeta c_{i-1}$, where ζ is given by (13). Thus, the collateral charged by bank $i + 1$ can be characterized recursively as

$$c_i = \min\{A_i, \zeta c_{i-1}\}, \quad (22)$$

with initial condition

$$c_1 = \min\{A_1, (1 - \phi_h)(1 - \phi_\ell)\Delta r k / \Delta\phi\}.$$

To solve (21), define the (weakly) decreasing sequence of numbers $\{k_0, k_1, k_2, \dots\}$ as

$$k_s = \min_{i \leq s} \frac{A_i \zeta^{1-i} (\Delta\phi / \Delta r)}{(1 - \phi_h)(1 - \phi_\ell)}, \quad (23)$$

with the convention $k_0 = \infty$. It is easy to verify that if $k \in [k_s, k_{s-1})$, bank s is the first bank whose collateral constraint binds; That is, $c_i < A_i$ for all $i < s$, whereas $c_s = A_s$. Now we separately maximize the objective function in (21) over the interval $k \in [k_s, k_{s-1})$ for all $s \in \{1, 2, \dots\}$.

Let $k \in [k_s, k_{s-1})$. As already mentioned, by construction, the collateral constraints of banks 1 through $s - 1$ do not bind. Recursion (22) thus implies that c_1, \dots, c_{s-1} are all proportional to k , whereas c_s, \dots, c_{n-1} do not depend on k , implying that the objective function in (21) is affine in k . Furthermore, this objective function is weakly increasing in k if and only if

$$\left(\frac{(1 - \phi_h)(1 - \phi_\ell)\Delta r}{\Delta\phi} \right) \left(\frac{1 - \phi_\ell + \alpha\phi_\ell}{\Delta\phi} \right) \sum_{j=1}^{s-1} \omega^{n-j-1} \zeta^{j-1} \geq \omega^n \Delta r - r_h.$$

Simplifying the above inequality leads to

$$\Delta r \omega^n [1 - (\zeta/\omega)^{s-1}] \geq \omega^n \Delta r - r_h,$$

which implies that

$$\left(\frac{\omega}{\zeta} \right)^s \geq \frac{\omega^n}{\zeta^{n^*}}, \quad (24)$$

where n^* is given by (14). Consequently, the maximum of the objective function in (21) is obtained at $k = k_{\underline{s}-1}$ where \underline{s} is the smallest integer for which inequality (24) is satisfied.

To summarize, if inequality (24) is satisfied for $s = 1$, then $k_{\max} = k_0 = \infty$. If, on the other hand, the smallest integer s that satisfies inequality (24) is greater than 1, then let

$$m = \left\lceil \frac{n \log \omega - n^* \log \zeta}{\log \omega - \log \zeta} \right\rceil - 1$$

which by assumption satisfies $m \geq 1$. Consequently, $\underline{s} = m + 1$ is the smallest integer that satisfies (24), implying that $k_{\max} = k_m$ defined in (23). Using (14) one more time completes the proof. \square

Proof of Corollary 2

From (5), we have

$$n^* - m = \left\lceil \frac{(n^* - n) \log \omega}{\log(\omega/\zeta)} \right\rceil.$$

On the other hand, recall from (14) that n^* is decreasing in Δr . Given that $\omega > \zeta$, it is immediate that $n^* - m$ is non-increasing in Δr . Thus, by equation (4), increasing Δr decreases k_{\max} . Similarly, it is immediate from the above equation that $n^* - m$ is decreasing in n . Therefore, by (4), increasing n reduces the intermediation capacity k_{\max} .

Finally, we show that k_{\max} is increasing in α . Note that we can rewrite (4) as

$$k_{\max} = \frac{A/\Delta r}{\omega(1 - \phi_h)\zeta^{m-1}}.$$

On the other hand, equation (5) implies that

$$(m - 1) \log \zeta = \log \zeta \left\lceil \frac{B}{\log(\omega/\zeta)} \right\rceil,$$

where B is a constant independent from α , and we are using the fact that $(n^* - 1) \log \zeta$ is independent of ζ . It is immediate to verify that the numerator and denominator of the right-hand side above are increasing and decreasing in ζ , respectively. Thus, $(m - 1) \log \zeta$ is increasing in ζ . On the other hand, from the proof of Proposition 1 we know that ζ is decreasing in α . Consequently, ζ^{m-1} is also decreasing in α , implying that k_{\max} increases with α . \square

Proof of Proposition 4

Denote the expected liquidation value of the asset conditional on the failure and the success of the inefficient project by \underline{z} and \bar{z} , respectively. It is thus immediate that $\phi_h \underline{z} + (1 - \phi_h) \bar{z} = 1$. Furthermore, one can show that

$$\begin{aligned} \bar{z} &= 1 + \rho\sigma \sqrt{\frac{\phi_h}{1 - \phi_h}} \\ \underline{z} &= 1 - \rho\sigma \sqrt{\frac{1 - \phi_h}{\phi_h}}. \end{aligned}$$

Proof of part (a) An argument identical to the proof of Proposition 1 implies that when banks are not collateral-constrained, then the chain efficiently intermediates the excess liquidity if and only if $n \leq n^*$ where n^* is the largest integer satisfying

$$\tilde{\zeta}^{n-1} \leq \frac{r_h \Delta \phi}{(1 - \phi_\ell) \Delta r},$$

in which

$$\tilde{\zeta} = 1 + \frac{(1 - \alpha) \phi_\ell (1 - \phi_h)}{\phi_h (1 - \phi_\ell) \underline{z} - \phi_\ell (1 - \phi_h)}. \quad (25)$$

It is immediate that the above expression is decreasing in \underline{z} , which in turn is decreasing in $\rho\sigma$. Thus, n^* is decreasing in $\rho\sigma$. \square

Proof of part (b) A recursive argument similar to that of Propositions 1 and 2 implies that the equilibrium collateral and interest rates charged by bank $i + 1$ to bank i are given by

$$c_i = (1 - \phi_h)\tilde{\omega}\tilde{\zeta}^{i-1}\Delta rk$$

and

$$R_i = r_h - \tilde{\omega}\phi_h z \tilde{\zeta}^{i-1}\Delta r,$$

respectively, where $\tilde{\zeta}$ is given by (25) and

$$\tilde{\omega} = \frac{1 - \phi_\ell}{\phi_h(1 - \phi_\ell)z - \phi_\ell(1 - \phi_h)}. \quad (26)$$

Note that both $\tilde{\omega}$ and $\tilde{\zeta}$ are decreasing in z , which in turn is decreasing in $\rho\sigma$. It is thus immediate that c_i is increasing in $\rho\sigma$. On the other hand, note that $\tilde{\omega}\phi_h z$ is decreasing in z , thus implying that R_i is increasing in z . Consequently, all interest rates are decreasing in $\rho\sigma$. \square

Proof of part (c) As in the proof of Proposition 3, suppose that bank n lends k units of capital to bank $n - 1$ which is then intermediated through the chain until bank 1 eventually invests it in the efficient project. Also, let m denote the first bank for which the collateral constraint binds, that is,

$$(1 - \phi_h)\tilde{\omega}\tilde{\zeta}^{m-1}\Delta rk \geq A > (1 - \phi_h)\tilde{\omega}\tilde{\zeta}^{m-2}\Delta rk. \quad (27)$$

Once again, an argument similar to the proof of Proposition 3 shows that the interest rate that bank n charges bank $n - 1$ is given by

$$R_{n-1} = r_h + \left(\frac{1 - \phi_\ell}{\Delta\phi}\right)^{n-m-1} \left[\frac{(1 - \phi_\ell + \alpha\phi_\ell)\Delta\phi + \phi_h(1 - \phi_\ell)(z - 1)}{(1 - \phi_h)\Delta\phi} (A/k) - (1 - \phi_\ell)\tilde{\zeta}^{m-1} \left(\frac{\Delta r}{\Delta\phi}\right) \right] - \left(\frac{\alpha\phi_\ell + \Delta\phi + \phi_h(z - 1)}{1 - \phi_h}\right) A/k.$$

Given that bank n 's opportunity cost of lending out k units of capital to the rest of the banking system is equal to $(1 - \phi_h)r_h k$, the intermediation capacity of the market is given by

$$k_{\max} = \arg \max_k \left\{ r_h k - (1 - \phi_\ell) \left(\frac{\Delta r}{\Delta\phi}\right) \omega^{n-m} \tilde{\zeta}^{m-1} k \right\},$$

where the domain over which the maximization is obtained is such that k and m satisfy (27). An argument similar to that of proof of Proposition 3 thus implies that m is given by

$$m = \left\lceil \frac{n \log \omega - n^* \log \tilde{\zeta}}{\log \omega - \log \tilde{\zeta}} \right\rceil - 1,$$

Thus, the intermediation capacity of the chain is given by

$$k_{\max} = \frac{A/\Delta r}{(1 - \phi_h)\tilde{\omega}\tilde{\zeta}^{m-1}}.$$

We can now perform the comparative statics of the above expression as a function of $\rho\sigma$. Recall from the proof of part (b) that $\tilde{\omega}$ and $\tilde{\zeta}$ are increasing in $\rho\sigma$. Furthermore, from the definition of m , it is immediate that m is also increasing in $\rho\sigma$, thus implying that k_{\max} decreases as $\rho\sigma$ is increased. \square

Proof of Proposition 6

Let s_i denote the number of banks in the i -th tier and (k_i, R_i, c_i) be the contract offered by an arbitrary bank in tier $i + 1$ to any given bank in tier i . To prove the proposition, we first show that the collection of contracts (k_i, R_i, c_i) given by recursions

$$k_i = \frac{k}{s_i s_{i+1}} \quad (28)$$

$$R_i = \zeta R_{i-1} - (\zeta - 1)r_h \quad (29)$$

$$c_i = \min \left\{ \frac{A}{s_{i+1}}, \zeta c_{i-1} \right\}, \quad (30)$$

constitute a symmetric equilibrium with no bank withdrawing its contract offers and all banks accepting all contracts offered to them.

To this end, consider a bank in tier $i + 1$ with excess cash equal to k/s_{i+1} contemplating what contract to offer to banks in tier i . Assuming that all other banks post contracts as prescribed by (28)–(30), it is immediate that this bank would only offer a loan of size $k_i = k/s_i s_{i+1}$ to all its potential borrowers in tier i . Note that if the bank offers a loan of any other size, there will be at least one bank on the i -th tier that ends up with excess cash strictly smaller than k/s_i , implying that it cannot fulfill its own contractual agreements to banks in tier $i - 1$. Given that such a bank will withdraw its contracts and hence, will end up investing in the inefficient project, the bank in tier $i + 1$ would never find it profitable to deviate from offering a loan of size k_i : the bank would always obtain a higher profit if it directly invests in the inefficient project itself. To summarize, conditional on offering a contract to banks in tier i , the best response contract of a bank in tier $i + 1$ has a loan size given by (28).

Next, we show that taking the contracts offered by other banks as given, the bank has no incentive to charge an interest rate or haircut distinct from (29) and (30). The problem faced by a bank in tier $i + 1$ is given by

$$\begin{aligned} \max_{\tilde{R}_i, \tilde{c}_i} \quad & (1 - \phi_\ell) \left(\frac{\tilde{R}k}{s_{i+1}} - \frac{R_{i+1}k}{s_{i+1}} \right) + \phi_\ell (\alpha s_i \tilde{c}_i - s_{i+2} c_{i+1}) \\ \text{s.t.} \quad & \tilde{\pi}_i \geq (1 - \phi_h)(r_h - R_i)k/s_i - \phi_h s_{i+1} c_i + A \\ & \tilde{\pi}_i \geq A \\ & \tilde{c}_i \leq A - (s_{i+1} - 1)c_i, \end{aligned}$$

where $c_i = 1 - k_i/(1 - h_i)$ is the collateral demanded by the bank and

$$\tilde{\pi}_i = (1 - \phi_\ell) \left(\frac{R_{i-1}k}{s_i} - \frac{\tilde{R}_i k}{s_i s_{i+1}} - \frac{R_i k (s_{i+1} - 1)}{s_i s_{i+1}} \right) + \phi_\ell (\alpha s_{i-1} c_{i-1} - \tilde{c}_i - (s_{i+1} - 1)c_i) + A.$$

is the expected profit of the borrower bank.¹⁹ A straightforward argument similar to the one in the proof of Proposition 2 shows that (28) and (29) are indeed the solutions to the above problem, thus establishing that contracts characterized via (28)–(30) constitute a symmetric equilibrium.

¹⁹Note that in the above problem, we are using the fact that the best response of the bank would be symmetric, in the sense that it has no incentive to offer different contracts to its different potential borrowers in tier i .

Now given the equilibrium contracts, an argument identical to the one in the proof of Proposition 3 implies that the intermediation capacity of the financial system is given by

$$k_{\max} = \min_{i \leq m} \frac{As_i \zeta^{n^* - i}}{r_h(1 - \phi_h)}$$

as long as $m \geq 1$, where m is given by (5), completing the proof. □

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