Insecure Debt*

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Abstract

We analyse bank runs under fundamental and liquidity asset risk, adopting a realistic description of bank default. A first result is that liquidity risk-driven runs occur frequently even as fundamental risk becomes arbitrarily small. The approach also enables to model the effect of pledging collateral to risk intolerant repo lenders. Safe repo debt is attractive as it is cheap and never runs, but increases risk for unsecured lenders, affecting their incentive to run. While a high rollover premium on unsecured debt could compensate for this, the private choice leads to more frequent runs. This direct form of risk creation is distinct from the externality effect caused by repo collateral sales in default, which lead to higher ex ante haircuts and further increase the frequency of inefficient runs.

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1 Introduction

During the recent US financial boom, credit expansion was boosted by strong demand for safe assets (Caballero and Krishnamurthy, 2009). As savers are historically willing to pay a safety premium, safe debt is cheap and thus a desirable source of funding (Krishnamurthy Vissing-Jorgensen, 2012). Banks and shadow banks responded by issuing safer liabilities, such as more short term commercial paper, as well as debt secured on financial collateral, known as *repo*. Once credit and liquidity risk became apparent, intermediaries suffered massive outflows of unsecured debt, which forced fire sales of illiquid assets. In contrast, repo credit raised haircuts but was rolled over, up to the eve of default (Gorton and Metrick, 2012; Krishnamurthy, Nagel and Orlov 2012). After Lehmann's default, unsecured lenders suffered heavy losses, while repo lenders were able to repossess and sell the pledged collateral. The induced fire sales played a critical role in propagating distress.

The experience of the crisis has led to sharper scrutiny of liquidity risk, such as the funding of illiquid securitized assets with short term debt and the role of secured financial credit (Duffie and Skeel, 2012).[1] In this paper we ask two questions. Next to fundamental risk, is there an effect of asset liquidity risk on runs? With hindsight, MBS prices fell way too low given real credit losses, as mortgage assets proved too illiquid to be backed by unstable funding. Second, how does the pledging of liquid assets to some lenders affects other funding sources? Secured funding proved very safe, and more stable than traditional unsecured funding. While its role in triggering collateral sales in default is now well recognized (Stein, 2012; Infante, 2013), its direct effect is not well understood.

We study these issues within a global game setting (Morris and Shin, 2003; Goldstein and Pauzner, 2005), under information on asset liquidity and fundamental risk. We show that asset liquidity risk causes a run equilibrium to arise, even if fundamental risk is arbitrarily small. In other words, in this setup almost all runs may be inefficient.[2]

[1] Next to repo, secured financial credit includes margins on derivative positions.

[2] In earlier models, when fundamental risk vanishes there is no unique run equilibrium, as the lower dominance region vanishes (Goldstein Pauzner, 2005).

Our results rely on a precise characterization of the default process in a bank bankruptcy. Traditional models assume all assets are sold immediately to satisfy withdrawals (Diamond and Dybvig, 1984). In reality, once a bank runs out of liquidity it is forced to declare default, triggering a mandatory stay on remaining unsecured lenders to allow orderly liquidation of illiquid assets.[3] Recognizing this feature leads to a simpler analysis of run incentives, as it restores equilibrium uniqueness.

Next we introduce secured debt as a funding choice. Provided enough safe collateral is pledged, repo is safe even in default, so it is always rolled over. The trade off is that unsecured debt now bears more risk, requiring a higher promised yield. Our contribution is to show that concentrating risk on unsecured debt increases the change of miscoordination via costly runs, unless compensated by a much higher yield. We show that the private choice of repo terms increases the frequency of runs: granting greater security to some lenders is temptingly cheap, but makes other lenders more insecure. A social planner will reduce the frequency of inessential runs by increasing the rollover premium above the private choice, leaving some rents to unsecured creditors.[4]

This direct effect adds to the liquidity risk externality associated with repo's fire sales of seized collateral upon default (Martin et al 2012, Oehmke 2014). The risk of collateral illiquidity induce safety-conscious repo lenders to adjust haircuts. We show how this increase in collateralization further concentrates risk on unsecured debt and thus shifts again the run threshold. Intermediaries recognize the run risk caused by repo borrowing, but as it is inexpensive they wish to attract as much as possible. The private funding choice tends to choose the maximum volume of secured debt and leaves no rents to unsecured debt to compensate for the chance of default. In conclusion, the private choice of repo debt results in more inefficient runs and default risk than the social optimum.

[3] Bankruptcy law was in fact introduced to solve the free rider problem when all creditors grab assets in an uncoordinated fashion, destroying value in the process.

[4] Note that in the absence of miscoordination, insuring risk intolerant lenders is efficient, and a strong safety premium reduce funding costs.

The sensitivity of the run threshold depends on asset liquidity and on the yield offered to demandable debt. A higher rollover premium reduce runs, so when repo is very cheap a social planner may boost the rollover yield to reduce runs. In contrast, a private intermediary will tend to minimize funding cost, choosing a higher average return at the cost of more runs. The effect of safe collateral on stability is interesting. More safe assets reduces the chance that the bank runs out of safe assets to repay withdrawals, which favors the rollover choice. However there is also an ambiguous rollover incentive effect, as full repayment also rises in a run, as well as the expected repayment for lenders who roll over. When collateral is low, the probability effect dominates, leading to fewer runs, while the latter is predominant when recovery in bankruptcy is high.

In sum, our result shows how secured debt may contribute to risk creation beyond the broadly recognized risk externality associated to repo's bankruptcy privileges (Duffie and Skeel, 2012). In our setting, more repo funding increases the frequency of (unsecured) runs, thus causing more collateral sales.[5] By itself, sales of repossessed collateral do not force faster liquidation of bank assets, as these are sold under orderly resolution. However, they reduce its liquidity, which in turn raises ex ante haircuts. So higher collateralization increases the chance of runs for all banks and reduce collateral liquidity, suggesting a risk externality not internalized by intermediaries. This effect does not arise in our framework, as the private choice of secured funding is already at the maximum.

In conclusion, unregulated secured funding does not just reallocate risk to those more willing to bear it, but adds risk by causing more runs and more early liquidation, a loss to be traded off against its lower cost. An implication is that regulatory policy should monitor and constrain the scale of secured funding, in order to reduce the associated liquidity risk.[6].

Finally, we show that increased instability caused by more repo funding creates larger losses for the deposit insurance fund. This occurs not only because pledged assets are subtracted in

[5]On the main features of safe harbor and the associated incentives to resell quickly seized collateral, see Perotti (2011) and Duffie and Skeel (2012)}.

[6]This may be achieved by capping it, or by imposing a Pigouvian charge to balance the externality effect. The optimal balance of ratio vs charges depends on the degree of bank capitalization (Perotti and Suarez, 2011)

default, but because of the increased run frequency.

A more complete welfare assessment of secured debt should take into account its role in satisfying the demand for safety by some investors, and thus expanding funding supply to credit intermediaries. We sidestep this issue by assuming that agents have a safe storage option, but the issue becomes salient in a situation of excess demand for safety. Attracting very risk averse agents at a low cost may increase the scale of investment, both by increasing the scale of funding and reducing the marginal required rate of return.[7]

Related literature

The literature explains repo funding in terms of a strong demand for absolute safety.[8] Existing work on repo credit does not compare it with debt of the same maturity, so there is no direct interaction effect. Martin, Skeie and von Thadden (2013) study the dynamics of repo runs, showing the critical role of collateral liquidity. Liquidity risk may trigger inefficient runs, analogous to runs driven by poor coordination by demandable debtors (Diamond and Dybvig (1983), Goldstein and Pauzner (2005)). He and Xiong (2011) provide a dynamic model of runs when debt is staggered, where creditors' roll-over decision depends on beliefs about other creditors' subsequent roll-over choice. Kuong (2013) considers the case when unsecured debt responds to higher repo margins by demanding higher required return, and shows that the resulting higher leverage directly affects risk taking by borrowers. Auh and Sundaresan (2014) looks at the effect of repo illiquidity risk on long term debt. In our set up, repo emerges as the preferred choice by investors seeking absolute safety. We assume all debt is demandable, or in any case has a common rollover date, to measure the interaction independently from maturity effect. In principle there should be a rationale for demandable debt, such as contingent liquidity demand by traditional depositors. Consistent with this view, intermediaries in our model hold some liquid reserves to meet withdrawals. Our results do not depend on secured debt being

[7]In a more complete model with decreasing returns, attracting repo funding enables marginal projects to be funded, as in Ahnert and Perotti (2014).

[8]This view is at the heart of recent work on shadow banking and safety traps (Gennaioli et al., 2013; Gorton and Ordoñez, 2014; Caballero and Fahri, 2013; Ahnert and Perotti, 2014.

demandable, as it is designed to be absolutely safe even in a run. Martin, Skeie and von Thadden (2013) propose that secured credit arise when asset values are non verifiable. Auh and Sundaresan (2014) argue that repo funding demands collateral to avoids violations of absolute priority.[9] They show that a bank may issue repo loans to save on the cost of long term debt, but will not issue too much when collateral liquidity is low.

Gorton and Ordoñez (2014) elaborate on the insight that information-insensitive claims arise to overcome adverse selection (Pennacchi and Gorton, 1999). Collateral runs may be triggered when it become information sensitive. Intriguingly, larger runs occur after a long positive period reduces the stock of public information.

A key driver in our approach is that safe claims are cheaper, as investors seeking absolute safety are willing to pay a safety premium. A financial pledge can be designed to avoid risk in all states. Such a strong investor preferences for safety has now been documented extensively (Gorton Lewellen Metrick (2012), Krishnamurthy Vissing-Jorgensen (2012)), and is leading to a new view of risk attitudes. Recent models assume some agents act as (locally) infinitely risk averse (Caballero Fahri (2013), Gennaioli et al (2013)). Alternatively, all agents may have Geary-Stone preferences demanding a subsistence level of wealth (Ahnert and Perotti (2014)).

Our direct effect of secured credit on stability complements the risk externality resulting from the special bankruptcy treatment for collateralized financial credit, the "safe harbor" privileges. This unique status creates a proprietary right directly enforceable on assets, and avoids risks such as excessive issuance or imperfect enforcement that may dilute a claim value. The ability of secured financial creditors to gain immediate access to the collateral is a unique privilege, as it grants superpriority status.

Legal scholars question whether it is justified to grant superior bankruptcy privileges to repo and derivatives (e.g. Morrison, Roe and Sontchi 2015). Bolton and Oehmke (2011) shows it leads to risk shifting incentives with derivatives. Duffie and Skeel (2012) argue that in order to reduce the risk of fire sales, only cash-like collateral may be excluded from automatic stay.[10]

[9] It may be ex post efficient to violate absolute priority, e.g. to ensure proper continuation incentives.

[10] This is equivalent to a narrow shadow banking model, also invoked in Gorton and Metrick (2012).

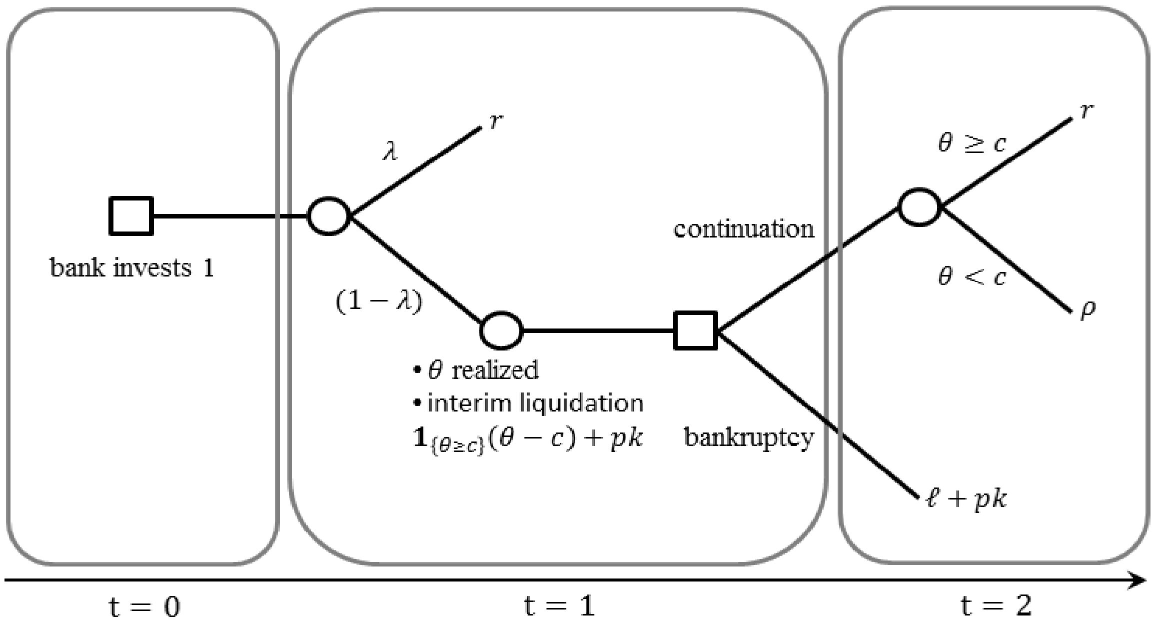
Limiting asset eligibility for safe harbor would limit the scale of encumbered assets and thus their direct effect on instability.

The consequences of immediate repossession of collateral became visible when Lehman Brothers collapsed. Within hours, hundred of billion in securities were repossessed, and immediately resold by repo lenders. The collapse in collateral prices and liquidity propagated the shock to the entire system. If repossession had occurred under a traditional orderly resolution plan, it would not have led to such rushed sales. Acharya, Anshuman and Vishwanthan (2012) argue for automatic stay provisions to avoid such fire sales. Perotti (2011) argues that safe harbor is what enables shadow banks to credibly promise liquidity on demand. By pledging the liquid component of assets, it replicates the banking model outside the regulated perimeter. Hanson, Stein, Shleifer and Vishny (2014) show how traditional banks are best at funding less risky but less liquid projects, while shadow banks promise liquidity by pledging liquid assets. In practice the distinction is not sharp, as repo funding issued by commercial banks is quite significant, also because of central bank refinancing. The degree of balance sheet encumbrance is thus a key stability question for banking supervisors.

2 The Basic Model

The economy lasts for three periods $t = 0, 1, 2$. It is populated by a bank and a continuum of lenders indexed by i. The intermediary has access to a project that needs one unit of funding in $t = 0$. It raises funds from lenders, each of whom is endowed with one unit. Lenders belong to one of two subsets. Some investors are risk neutral and demand a minimum expected return of $\gamma > 1$, reflecting their alternative option. A set of investors is infinitely risk averse, willing to lend if and only if they can be assured to be paid in full in all states. In exchange for absolute safety, they accept a lower return equal to 1, the rate that they could earn on safe storage. The measure of each subset is sufficiently large such that the bank could in principle finance the project with only one type of lender.

Figure 1: Project Timeline



- *Project*

For each unit invested, the project generates a return of $y_t(\omega)$ in $t = 1, 2$, where $\omega \in \{H, L\}$ is the aggregate state. With probability λ the state is revealed at $t = 1$ to be high ($\omega = H$), and the project matures in $t = 1$: $y_1(H) = r > \gamma$. With probability $1 - \lambda$, the state is revealed to be low ($\omega = L$), and the project matures only in $t = 2$. In this case, early liquidation at $t = 1$ could be costly as the project has not fully developed its potential. The early liquidation value has a safe component $k > 0$ plus an uncertain value θ, drawn from a uniform distribution on $\left[0, \overline{\theta}\right]$. Liquidating risky assets inolves a fixed cost c. All agents receive private signals on θ at the begin of time 1. In the low state there is some fundamental risk at time 2. Asset returns are the same as in the high state ($y_2(L) = r$) as long as $\theta \geq c > 0$; however, if $\theta < c$, the return is only $y_2(L) = \rho < 1$. As long as c is small, this assumption says that the project is almost riskless if allowed to mature, except in states when the interim liquidation value is also very low.

A claim to the safe portion of the return may be securitized in order to be pledged to lenders seeking higher safety. If sold at time 1, financial collateral returns an ex ante known price $p \in (0, 1]$, where the discount reflects limited interim liquidity. To ensure that depositors can be fully repaid in the low state for a sufficiently high liquidation value, we assume that $\overline{\theta} + pk > 1 + c$. This assumption and the dependence of $y_2(L)$ on θ create an upper and a lower dominance region in our global game setup, needed to ensure equilibrium uniqueness.

- *Lenders and Financing*

The bank raises funds by issuing a measure s of secure debt, and the residual $1 - s$ through unsecured debt with face value d. We will refer to them by the subscripts $\{U, S\}$. The project has a positive NPV for any funding choice. However, in order to attract risk intolerant investors the bank needs to offer an absolutely safe claim. Specifically, the bank can securitize the safe part of its return and pledges some of it as collateral to secured lenders.

We assume (realistically) that a fraction of financial collateral $k - \underline{k} > 0$ must be retained by the bank, so the maximum amount that can be pledged to back secured debt is $\underline{k}$. An interpretation is a minimum reserve requirement (such as the Liquidity Coverage Ratio under Basel III), to ensure some liquidity to meet routine withdrawals.

Because financial collateral is sold at a discount at time 1 (at price $p \leq 1$), secured lenders at $t = 0$ demand an adequate haircut h — excess collateral per unit of funding — to be sure of full repayment at $t = 1$. Provided the haircut satisfies $hp \geq 1$, secured lenders are always paid in full even in default, and never face losses. As a result, unsecured debt in default has access to $k - hs$ units of financial collateral. Since a higher s reduces the amount of safe return available to unsecured debt, it decreases the probability that the bank survives in the event of a large run. Thus for a given run probability, unsecured creditors will require a higher face value to compensate for larger losses in a run.

- *Lenders' Information Structure*

All agents observe the state ω at the begin of $t = 1$. In the high state the project has matured so all claims are safe. In the event of a low state $\omega = L$, agents receive individual noisy signals on the early liquidation value of assets $\theta + k$.[11] This signal is given by

$$x_i = \theta + \sigma \eta_i, \tag{1}$$

where $\sigma > 0$ is an arbitrarily small scale parameter and η_i are i.i.d. across players and uniformly distributed over $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

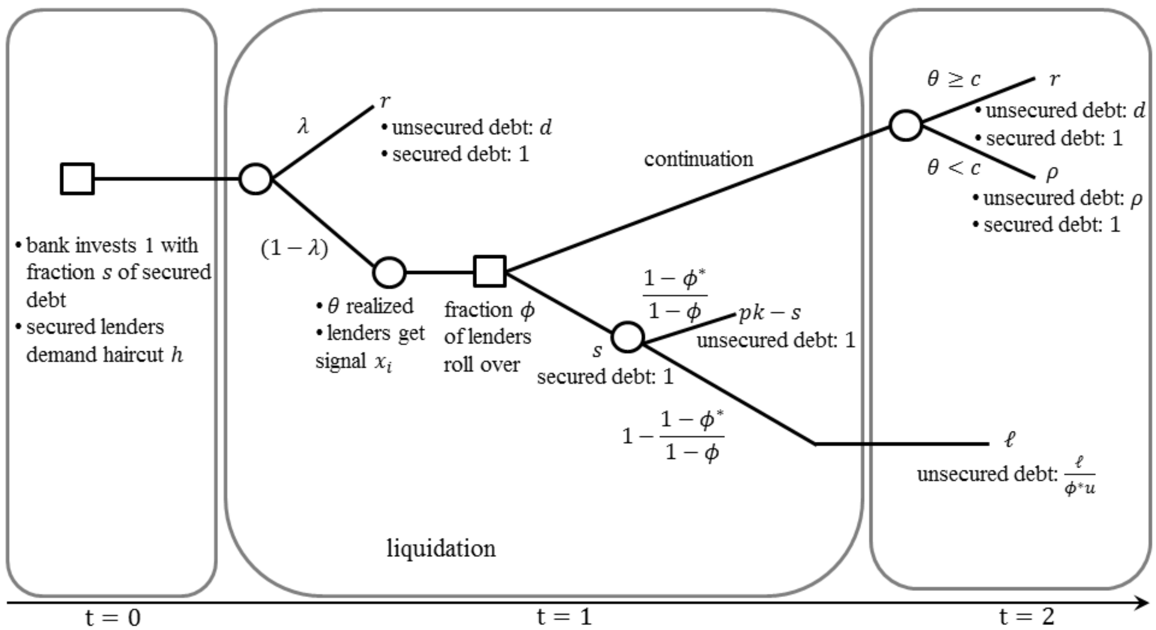
- *Debt Rollover, Bank Runs, and Orderly Liquidation*

Since all claims are safe in the high state, we henceforth focus on the low state $\omega = L$. By design, secured agents are fully protected even in default, and have no incentive to run. In contrast, upon receiving their private signal unsecured lenders may choose to withdraw the principal amount 1. In this case, the bank sells assets sequentially in order to meet withdrawals, starting from its reserves of (unpledged) liquid collateral. Thus the first agents to withdraw are ensured of repayment. If withdrawals exceed the value of liquid reserves, the bank needs to liquidate a part of the real project. If the remaining withdrawals are larger than the maximum liquidation proceeds minus the fixed cost of c, the bank is immediately declared in default and the early liquidation process is halted. This is a realistic description of bank bankruptcy. Once a bank is declared in default, bankruptcy law forces a stay for all unsecured creditors, avoiding fire sales and the fixed cost c and enabling orderly resolution at $t = 2$. At that point, any unpaid depositors are treated equally. This differ from the standard assumption that withdrawals are met by selling all assets immediately, which is clearly less efficient. The bankruptcy process produces a final excess value equal to $\ell \geq 0$ under orderly liquidation.

We assume that even if the bank goes bankrupt with certainty in the low state, the project has positive NPV if fully funded by risk neutral investors: $\lambda r + (1 - \lambda)(pk + \ell) - \gamma > 0$. This allows us to focus on the main tradeoff of the problem without imposing addtional constraints to financing. We also asume that $r - 1 \leq 1 - \ell - pk$. This assumption ensures equilibrium uniqueness and has two natural interpretations. First, it ensures that the value produced under

[11] To be precise, θ is the liquidation value of the asset in excess of the safe component.

Figure 2: Economy Timeline



orderly liquidation is never enough to fully repay all unsecured lenders: $\ell + pk < 1$. Second, it suggests that higher project return (higher r) must be associated with higher risk (lower $\ell + pk$).

The bankruptcy event is formally characterized as follows. Let ϕ be the fraction of unsecured lenders that roll over in $t = 1$. The first depositors in the running queue are paid out of liquid reserves (the retained collateral $k - sh$). If there are depositors left in the queue, the bank is declared bankrupt if and only if

$$u\,(1-\phi) > \theta - c + p\,(k - sh)\,.$$

Here the left hand term indicates the face value demanded by running depositors, and the right hand side the amount available at $t = 1$, namely the net liquidation value of the unencumbered asset value plus the value of the retained collateral. The condition may be rewritten as stating that the bank is declared bankrupt if the net value of selling its non reserve assets exceeds the claims of the still unpaid withdrawing depositors.

- *Lenders' Payoffs*

When the state is high, unsecured lenders always receive their face value d. In the low state, their payoffs depend on whether they choose to roll over or withdraw, and whether the bank survives in case of a run. Early withdrawals receive the principal 1 if the bank has enough liquidity. If it defaults, liquidation proceeds net of pledged assets are distributed equally among all unpaid unsecured debt holders.

In a run, the random order of arrival implies that running depositors receive their full principal out of the liquid reserves with probability $\frac{1-\phi^*}{1-\phi}$, where ϕ^* is such that $(1-\phi^*)\,u = p\,(k-sh)$. That is, ϕ^* is the minimum fraction of unsecured lenders that needs to roll over in order for all withdrawers to receive full repayment out of reserves. With probability $1-\frac{1-\phi^*}{1-\phi}$, withdrawers receive $\frac{\ell}{\phi^* u}$, the orderly liquidation value of unencumbered assets scaled by the mass of remaining unsecured lenders.

In conclusion, the payoff of unsecured lenders who do not roll over in $t=1$ is

$$\pi_U^N(\phi,\theta) = \begin{cases} 1, & \text{if } u\,(1-\phi) \leq \mathbf{1}_{\{\theta\geq c\}}\,(\theta-c)+p\,(k-sh) \\ \frac{1-\phi^*}{1-\phi}+\left(1-\frac{1-\phi^*}{1-\phi}\right)\frac{\ell}{\phi^* u}, & \text{if } u\,(1-\phi) > \mathbf{1}_{\{\theta\geq c\}}\,(\theta-c)+p\,(k-sh) \end{cases},$$

where $\mathbf{1}_{\{.\}}$ is the indicator function.

Unsecured lenders that roll over receive the face value of their loans if the bank does not go bankrupt. In bankruptcy, they are entitled to receive $\frac{\ell}{\phi^* u}$ out of the orderly liquidation value. That is,

$$\pi_U^R(\phi,\theta) = \begin{cases} \mathbf{1}_{\{\theta\geq c\}}d+\left(1-\mathbf{1}_{\{\theta\geq c\}}\right)\rho, & \text{if } u\,(1-\phi) \leq \mathbf{1}_{\{\theta\geq c\}}\,(\theta-c)+p\,(k-sh) \\ \frac{\ell}{\phi^* u}, & \text{if } u\,(1-\phi) > \mathbf{1}_{\{\theta\geq c\}}\,(\theta-c)+p\,(k-sh) \end{cases}.$$

Notice that this payoff is decreasing in both s and h.

3 Insecure Debt Runs without Repo

We begin by analyzing the model in the absence of secured debt ($s = 0$). This is useful to establish a benchmark to analyze the effects of introducing secured funding. It also enables to show the effect of a fuller modelling of the bankruptcy process on general run equilibria.

The financing contract is now fully characterized by the face value of unsecured debt d.

3.1 Equilibrium Runs

The bank is assessed to be bankrupt if and only if withdrawals $1-\phi$ are sufficiently large:

$$(1-\phi)\,u > \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + pk = \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + pk, \tag{2}$$

Let $\Pi_U^R\,(\phi, \theta)$ be the net payoff of unsecured lenders who roll over relative to that of running. We have

$$\Pi_U^R\,(\phi, \theta) = \begin{cases} \mathbf{1}_{\{\theta \geq c\}} d + \left(1 - \mathbf{1}_{\{\theta \geq c\}}\right) \rho - 1, & \text{if } u\,(1-\phi) \leq \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + pk \\ -\frac{pk}{(1-\phi)} \left(1 - \frac{\ell}{1-pk}\right), & \text{if } u\,(1-\phi) > \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + pk \end{cases}. \tag{3}$$

Since unsecured lenders require a minimum expected return of $\gamma > 1$, the face value of unsecured debt d must be larger than 1. As a result, unsecured lenders face a complex coordination problem in their decision to roll over, which depends on their beliefs about both θ (fundamental uncertainty) and the fraction ϕ of lenders that rolls over (strategic uncertainty).

Suppose lenders follow a monotone strategy with a cutoff κ, rolling over if their signal is above κ and withdraw otherwise. Lender i's expectation about the fraction of lenders that roll over conditional on θ is simply the probability that any lender observes a signal above κ, that is, $1 - \frac{\kappa - \theta}{\sigma}$. This proportion is less than z if $\theta \leq \kappa - \sigma\,(1-z)$. Each lender i calculates this probability using the estimated distribution of θ conditional on his signal x_i.

We rely now on the well known result in the literature of global games that as $\sigma \to 0$, this probability equals z for $x_i = \kappa$.[12] That is, the threshold type believes that the proportion of lenders that roll over follows the uniform distribution on the unit interval. Focusing on the situation when signals become nearly precise enables to highlight strategic uncertainty rather than uncertainty about θ. The equilibrium cutoff can then be computed by the threshold type who must be indifferent between rolling over and withdrawing given his beliefs about ϕ.

Let θ^* be such cutoff. Since $\Pi_U^R(\phi, \theta)$ is always negative for $\theta < c$, θ^* is greater than c and is given by the unique solution to

$$\int_0^{1-(\theta^*-c+pk)} \left[-\frac{pk}{(1-\phi)}\left(1-\frac{\ell}{1-pk}\right)\right] d\phi + \int_{1-(\theta^*-c+pk)}^{1} (d-1)\, d\phi = 0. \tag{4}$$

This leads us to Proposition 1.

Proposition 1 (Run Cutoff without Repo) *In the limit $\sigma \to 0$, the unique equilibrium in $t = 1$ has unsecured lenders following monotone strategies with threshold θ^* given by*

$$\theta^* = e^{-W\left(\frac{d-1}{pk\left(1-\frac{\ell}{1-pk}\right)}\right)} + c - pk, \tag{5}$$

where all unsecured lenders roll over if $\theta > \theta^*$ and do not roll over if $\theta < \theta^*$.[13]

Proposition 1 allows us to derive how the probability of bankruptcy relates to the bank's financing policy and the financial collateral. Recall that a lower θ^* is desirable, as it implies less frequent runs.

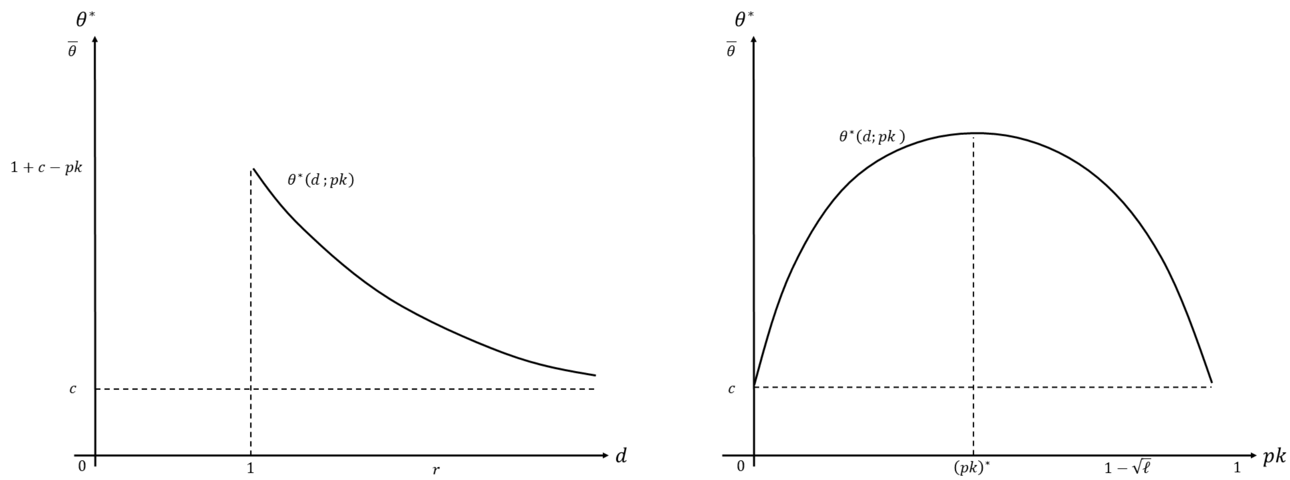
Corollary 1 (Yield and Collateral Effects on Stability) *θ^* has the following properties:*

(i) It is decreasing and concave in the roll over premium d, and concave in collateral value pk: $\frac{\partial\theta^*}{\partial d} < 0$, $\frac{\partial^2\theta^*}{\partial d^2} > 0$, and $\frac{\partial^2\theta^*}{\partial (pk)^2} < 0$.

(ii) There exists a cutoff $(pk)^* \in \left(0, 1-\sqrt{\ell}\right)$ such that θ^* is strictly decreasing in pk for $pk \geq (pk)^*$ and is strictly increasing in pk for $pk < (pk)^*$.

[12] See Morris and Shin (2003) for a comprehensive discussion of the global games literature.
[13] $W(\cdot)$ is known as the Lambert W function and is the inverse function of $y = xe^x$ for $x \geq -1$.

Figure 3: Run Cutoff without Repo



(a) Function of Unsecured Yield (b) Function of Collateral

Corollary 1 can be more easily interpreted after rewriting (15):

$$\underbrace{\theta^* - c + pk}_{\text{unsecured debt recovery ratio}} = e^{-W\overbrace{\left(\frac{d-1}{pk\left(1-\frac{\ell}{1-pk}\right)}\right)}^{\text{rolling over net-benefit-to-cost ratio}}}. \tag{6}$$

From (16), the signal θ^* makes the threshold unsecured lender type just indifferent, balancing the recovery ratio in a run against the rollover premium $d - 1$. The first term (the unsecured debt recovery ratio) captures a "probability" effect. That is, it measures the likelihood that the bank has enough resources to repay unsecured lenders in a run, which depends on the realized θ and the stock of unpledged collateral. The second term contains the rollover net-benefit-to cost ratio, which measures a "relative payoff" effect, the net benefits of rolling over when there is no bankruptcy relative to the losses incurred in a run.

The results of Corollary 1 (illustrated in Figure 2) offer some insight on the comparative statics of the equilibrium run threshold θ^* and thus the frequency of runs. The effect of the face value of unsecured debt d on θ^* is intuitive. Increasing it improves the payoff of rolling over for a given chance of default, and unambiguously reduces the probability of runs. However, promising a large reward to unsecured lenders comes at the cost of the return to the bank in all solvent states. This observation is important to understand the bank's pricing choice.

An increase in the liquidation value of the financial collateral pk affects the run threshold in two ways. It has an unambiguous linear "probablity" effect, reducing the chance that the bank runs out of safe assets to repay withdrawals. This effect therefore contributes to a lower cutoff θ^*. It also has an ambiguous "relative payoff" effect. First, it increases the chance that withdrawals are fully paid out of safe assets in the event of bankruptcy. Second, it reduces the amount of withdrawals not paid out of the financial collateral, increasing the likelyhood that lenders who roll over are fully paid in bankruptcy out of the orderly liquidation value ℓ . While the former stimulates runs and contributes to a higher θ^*, the latter reduces the opportunity cost of rolling over and decreases θ^*. Corollary 1 shows that the former effect dominates when the value of the collateral is low and the latter is predominant when recovery in bankruptcy is high.

Proposition 1 also allows us to derive important results concerning the inefficiency of runs. It implies that runs are efficient if and only if: (i) the bank is insolvent, $0 \leq \theta < c$, and (ii) the project return in the event of insolvency is no greater than the bankruptcy proceeds, $\rho \leq pk + \ell$. This is formalized in Corollary 2:

Corollary 2 *All runs are inefficient if $\rho > pk + \ell$. If $\rho \leq pk + \ell$, then almost all runs are inefficient for $c \to 0$, in which case the probability of runs is bounded away from zero: $\theta^* \geq e^{-W\left(\frac{1-pk}{pk}\right)} - pk > 0$.*

Corollary 2 also states that, even if almost all runs are inessential, the probability of runs is not negligible.

3.2 The Optimal Funding Choice

This section examines the bank's initial funding choice d. Because the project has positive NPV for any funding choice, we can focus on the stability tradeoff, excluding other effects of its financing structure.

The ex ante expected payoff of unsecured lenders as a function of its face value d is

$$V_U(d) = \lambda d + (1-\lambda)\left[\frac{\overline{\theta} - \theta^*(d)}{\overline{\theta}} d + \frac{\theta^*(d)}{\overline{\theta}}(pk + \ell)\right]$$

The bank's expected payoff can be written as the return of the project of a solvent bank r net of financing costs and the expected deadweight loss $DW(d)$:

$$\begin{aligned} V_B(d) &= \lambda(r-d) + (1-\lambda)\left(\frac{\overline{\theta} - \theta^*(d)}{\overline{\theta}}\right)(r-d) \\ &= r - V_U(d) - DW(d), \end{aligned} \tag{7}$$

where $DW(d)$ is the total payoff lost in the event of bankruptcy, that is

$$DW(d) = (1-\lambda)\frac{\theta^*(d)}{\overline{\theta}}(r - pk - \ell). \tag{8}$$

3.2.1 Socially Optimal Funding

As a benchmark, we characterize the optimal financing contract chosen by a social planner. The social planner chooses the face value of unsecured debt d that maximizes the aggregate payoff subject to the participation constraint of the bank and unsecured lenders:

$$\max_d \; r - DW(d) \tag{9}$$

subject to

$$V_B(d) \geq 0, \; V_U(d) \geq \gamma.$$

In other words, the optimal financing policy minimizes the chance of runs (a deadweight loss) subject to agents' participation constraints. Since $-DW(d)$ is increasing in d, the social planner would like to increase d as much as possible.

To pin down the solution, we use our assumption that r is sufficiently large. Increasing d relaxes the lenders' participation constraint. However, the bank's participation constraint is binding at $d = r$, when all asset value is promised to depositors. In addition, the bank's payoff is concave and decreasing at $d = r$, which implies that this is the maximum face value that may be chosen by the social planner. Since the bank's participation constraint binds, it follows that the lenders' participation constraint does not bind. Proposition 2 characterizes the socially optimal financing policy.

Proposition 2 (Optimal Funding without Repo) *The socially optimal financing contract requires the bank to issue unsecured debt with the maximum possible face value* $d^o = r$*.*

Intuitively, the social planner care about minimizing losses due to early withdrawals, and thus boosts the incentive to roll over by offering teh maximum possible roll over yield.

3.2.2 Private Funding Choice

The bank's problem is to choose the return to unsecured debt d that maximize its payoff subject to the participation constraint:

$$\max_{d} V_B(d) \tag{10}$$

$$\text{subject to}$$

$$V_U(d) \geq \gamma.$$

In choosing its optimal funding structure, the bank faces a tradeoff between the cost of financing and the expected deadweight loss. The cost of financing is increasing in the face value of unsecured debt d. However, lower d makes runs more likely, which increases the expected deadweight loss.

Proposition 3 (Private Inefficiency without Repo) *The probability of bankruptcy under the socially optimal funding structure is always lower than under the private funding choice:*

$\theta^* (d^o) < \theta^* (d^*)$.

While the social planner minimizes the probability of runs by choosing the maximum feasible face value of unsecured debt $d^o = r$, the bank trades off the probability of bankruptcy against the cost of debt. As a result,, the private choice of d^* is lower than the social optimum value, leading to a higher threshold $\theta^* (d^*) > \theta^* (d^o)$ and thus more frequent runs.

Proposition 4 characterizes the optimal private funding choice.

Proposition 4 (Private Funding without Repo) *The bank's financing policy is characterized as follows:*

(i) The optimal face value of unsecured debt d^* is characterized by either $\mu^* [V_U (d^*) - \gamma] = 0$ or $-\frac{\partial DW(d^*)}{\partial d} = \frac{\partial V_U(d^*)}{\partial d} [1 - \mu^*]$, where μ^* is the Lagrange multiplier associated with unsecured lenders' participation constraint.

(ii) There exists a cutoff $\lambda_1 \in [0, 1)$ such that, if $\lambda > \lambda_1$, unsecured lenders' participation constraint binds, i.e., $V_U (d^*) - \gamma = 0$.

The face value of unsecured debt balances the lower cost of funding against the higher expected deadweight loss from reducing d subject to the participation constraint. The condition for d^* is sufficient as we show that $V_B (d)$ is concave. When λ is sufficiently high the optimal funding structure is a corner solution as in this case the possibility of runs is rare.

4 Insecure Debt Runs with Repo

We now examine the effect of allowing the bank to borrow from secured lenders. As we will see, although secured debt can be made so safe that it never runs, it concentrates risk on unsecured lenders, potentially triggering more runs. Thus, secured debt not only redistributes but also increases risk.

4.1 Equilibrium Runs

In order to derive the optimal rollover decision, we first calculate the haircut h demanded by secured lenders. The haircut are set at $t = 0$ to make sure they are paid in full in the event of a run at $t = 1$. Recall that the sale price of financial collateral p is known as of time 0.

The payoff of each secured lenders in $t = 1$ in case of a run is $\pi_S = ph$. Therefore, the minimum haircut h^* demanded by unsecured lenders in $t = 0$ solves

$$ph^* = 1. \tag{11}$$

If the bank pledges sh^* units of financial collateral, secured lenders are completely safe as long as $sh^* = sp^{-1} \leq \underline{k}$, which imposes a higher bound on the volume of secured debt $s \leq p\underline{k}$.

Consider now the unsecured lenders' rollover decision. From the previous section, the bank is assessed to be bankrupt if and only if withdrawals $1 - \phi$ are sufficiently large:

$$(1-\phi)\,u > \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + p\,(k - sh^*) = \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + pk - s, \tag{12}$$

Let $\Pi_U^R(\phi, \theta)$ be the net payoff of unsecured lenders who roll over relative to that of running. We have

$$\Pi_U^R(\phi,\theta) = \begin{cases} \mathbf{1}_{\{\theta \geq c\}} d + \left(1 - \mathbf{1}_{\{\theta \geq c\}}\right)\rho - 1, & \text{if } u\,(1-\phi) \leq \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + p\,(k - sh) \\ -\frac{pk-s}{(1-\phi)(1-s)}\left(1 - \frac{\ell}{1-pk}\right), & \text{if } u\,(1-\phi) > \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + p\,(k - sh) \end{cases}. \tag{13}$$

Since unsecured lenders require a minimum expected return of $\gamma > 1$, the face value of unsecured debt d must be larger than 1, its maximum early withdrawal value. As in the previous section, we focus on strategic uncertainty as signals become nearly precise. The equilibrium cutoff is computed using the condition that the threshold type must be indifferent between rolling over and withdrawing given his uniform beliefs about ϕ.

Let θ^* be such cutoff. Since $\Pi_U^R(\phi, \theta)$ is always negative for $\theta < c$, θ^* is greater than c and is given by the unique solution to

$$\int_0^{1-\frac{\theta^*-c+pk-s}{1-s}} \left[-\frac{pk-s}{(1-\phi)(1-s)} \left(1 - \frac{\ell}{1-pk} \right) \right] d\phi + \int_{1-\frac{\theta^*-c+pk-s}{1-s}}^{1} (d-1)\, d\phi = 0. \tag{14}$$

This leads us to Proposition 5.

Proposition 5 (Run Cutoff with Repo) *In the limit $\sigma \to 0$, the unique equilibrium in $t = 1$ has unsecured lenders following monotone strategies with threshold θ^* given by*

$$\theta^* = (1-s)\, e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\right)} + c - (pk - s)\,, \tag{15}$$

where all unsecured lenders roll over if $\theta > \theta^*$ and do not roll over if $\theta < \theta^*$.[14]

Proposition 5 allows us to derive the relation between the probability of bankruptcy and secured credit:

Corollary 3 (Direct Repo Effects on Stability) *θ^* has the following properties:*

(i) $\frac{\partial^2 \theta^*}{\partial s^2} < 0$.

(ii) For $s \in [0, pk)$, θ^* is strictly decreasing in s for $d \leq \underline{d}(0)$, and is first strictly increasing, then strictly decreasing in s for $d > \underline{d}(0)$, where $\underline{d}(0) = 1$ if and only if $pk \geq \frac{1}{2}$.

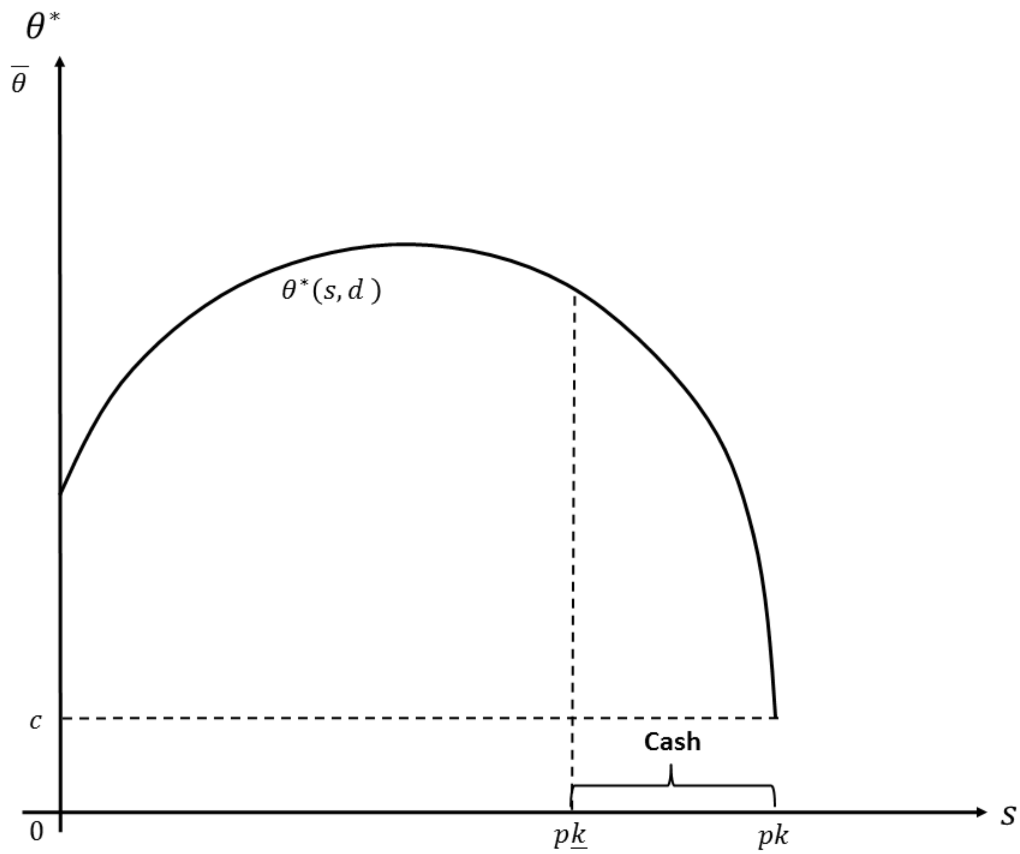
The results of Corollary 3 can be more easily interpreted after rewriting (15):

$$\underbrace{\frac{\theta^* - c + (pk - s)}{1-s}}_{\text{unsecured debt recovery ratio}} = e^{-W \overbrace{\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\right)}^{\text{rolling over net-benefit-to-cost ratio}}}. \tag{16}$$

The effect of secured debt s on θ^* is concave, while the sign of its derivative is ambiguous. More secured debt reduces the amount of collateral retained and available to withdrawers,

[14] $W(\cdot)$ is known as the Lambert W function and is the inverse function of $y = xe^x$ for $x \geq -1$.

Figure 4: Run Cutoff with Repo



decreasing the opportunity cost of rolling over. However, it also reduces the recovery ratio of unsecured debt, making the bank more likely to go bankrupt for any given fraction of funds withdrawn. The slope of the threshold curve in s at $s = 0$ depends on the rollover premium d. Since d needs to be above 1, the Corollary indicates that whenever asset liquidation risk is low (k is large, so that $pk \geq \frac{1+s}{2}$), a small increase in secured debt above $s = 0$ always leads to a higher risk of runs. In this case the threshold θ^* will be at first rising and then falling in the amount of secured debt. Figure 3 shows such a case when the probability of runs is first increasing in s, reflecting the dominance of the probability effect for lower levels of secured debt, then decreasing when the payoff effect becomes more prominent.[15] As d is set high (closer to the participation constraint of the bank), $\theta^*(s, d)$ shifts lower. Its intercept

[15] Both $\frac{\theta^* - c + (pk - s)}{(1-s)}$ (the probability effect) and $e^{-W(s)}$ (the payoff effect) are decreasing and concave in s. However, as s goes to pk, the derivative of $\frac{\theta^* - c + (pk - s)}{(1-s)}$ converges to a finite number while that of $e^{-W(s)}$ goes to minus infinity. Therefore, for s large, the payoff effect dominates the probability effect.

value is lower (and thus lower is the run risk) at $s = 0$ than at the maximum amount possible $s = p\underline{k}$. The threshold $\theta^*(s, d)$ may also be downward sloping from $s = 0$ when the rollover reward offered to unsecured debt d is very low and the asset liquidation value is risky (that is, k is sufficiently low).

The direct effects of the face value of unsecured debt and the financial collateral on the run cutoff θ^* are qualitatively the same as those encountered in the previous section, when secured debt funding was not allowed. Let us turn our analysis to the interaction effects of repo funding, unsecured yield, and financial collateral on bank runs.

Corollary 4 (Indirect Repo Effects on Stability) *θ^* has the following properties:*

(i) There exists a cutoff $\underline{d}(s) \geq 1$ such that $\frac{\partial^2 \theta^*}{\partial s \partial d} \leq 0$ for $d \leq \underline{d}(s)$ and $\frac{\partial^2 \theta^*}{\partial s \partial d} > 0$ for $d > \underline{d}(s)$, where $\underline{d}(s) = 1$ if and only if $pk \geq \frac{1+s}{2}$.

(ii) There exists a cutoff $(pk)^* \in \left(s, 1 - \sqrt{\ell(1-s)}\right)$ such that θ^* is strictly decreasing in pk for $pk \geq (pk)^*$ and is strictly increasing in pk for $pk < (pk)^*$, where $(pk)^*$ is strictly increasing in s.

(iii) If $(1 - pk)^2 \leq \ell$, there exists a cutoff $\underline{s} \in \left(\frac{\ell - (1-pk)^2}{\ell}, pk\right)$ such that θ^* is strictly decreasing in pk for $s \leq \underline{s}$ and is strictly increasing in pk for $s > \underline{s}$.

4.2 Funding

This section examines the bank's initial funding choice (s, d). Because the project has positive NPV for any funding choice, we can focus on the stability tradeoff, excluding other effects of its financing structure.

The ex ante expected payoff of unsecured lenders as a function of its face value d is

$$V_U(s, d) = \lambda d + (1 - \lambda)\left[\left(\frac{\overline{\theta} - \theta^*(s, d)}{\overline{\theta}}\right) d + \frac{\theta^*(s, d)}{\overline{\theta}} \frac{pk - s + \ell}{1 - s}\right] \tag{17}$$

The bank's expected payoff can be written as the return of the project of a solvent bank r net of financing costs and the expected deadweight loss $DW\left(s,d\right)$:

$$V_B\left(s,d\right) = \lambda\left[r - d\left(1-s\right) - s\right] + \left(1-\lambda\right)\left(\frac{\overline{\theta} - \theta^*\left(s,d\right)}{\overline{\theta}}\right)\left[r - d\left(1-s\right) - s\right] \tag{18}$$
$$= r - s - \left(1-s\right)V_U\left(s,d\right) - DW\left(s,d\right),$$

where $DW\left(s,d\right)$ is the total payoff lost in the event of bankruptcy, that is

$$DW\left(s,d\right) = \left(1-\lambda\right)\frac{\theta^*\left(s,d\right)}{\overline{\theta}}\left(r - pk - \ell\right). \tag{19}$$

4.2.1 Socially Optimal Funding

As a benchmark, we characterize the optimal financing contract chosen by a social planner. The social planner chooses a pair $\left(s,d\right)$ that maximizes the aggregate payoff subject to the participation constraint of the bank and unsecured lenders:

$$\max_{s,\,d}\; r - DW\left(s,d\right) \tag{20}$$

subject to

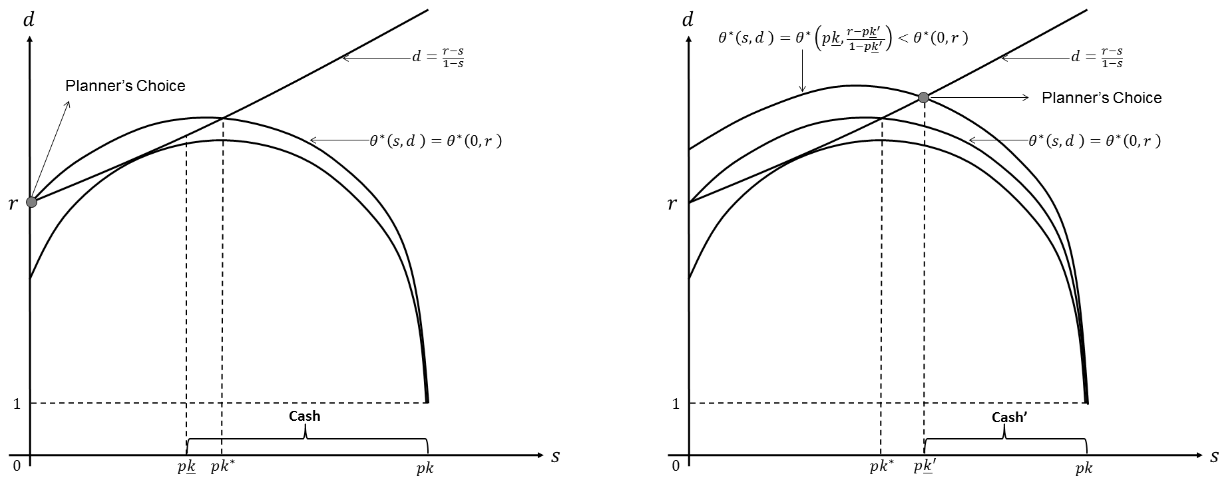
$$V_B\left(s,d\right) \geq 0,\; V_U\left(s,d\right) \geq \gamma,\; s \in \left[0, p\underline{k}\right].$$

In other words, the optimal financing policy minimizes the deadweight loss subject to agents' participation constraints. Since $-DW\left(s,d\right)$ is increasing in d, the social planner would like to increase d as much as possible for any fraction of secured debt s.

Increasing d relaxes the lenders' participation constraint. However, the bank's participation constraint is binding at $d = \frac{r-s}{1-s}$. In addition, the bank's payoff is concave and decreasing at $d = \frac{r-s}{1-s}$, which implies that this is the maximum face value that can be chosen by the social planner.

To pin down the solution to the planner's problem, we make use of our assumption that r

Figure 5: Socially Optimal Funding



(a) No Secured Debt (b) Maximum Secured Debt

is sufficiently large. Since the bank's participation constraint binds, it follows that the lenders' participation constraint does not bind. Moreover, since $d = \frac{r-s}{1-s} > r$ and θ^* is increasing in s for d sufficiently large, the deadweight loss is increasing in the fraction of secured debt. This is illustrated in Figure 3. Therefore, the social planner's optimal choice is to set $s = 0$, which results in a face value of $d = r$. Proposition 6 characterizes the socially optimal financing policy.

Proposition 6 (Optimal Funding with Repo) *The socially optimal financing contract* (s^o, d^o) *requires the bank to issue either only unsecured debt* $(s^o, d^o) = (0, r)$*, or the maximum possible amount of secured debt* $(s^o, d^o) = \left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$*. There exists a cutoff* $k^* \in (0, k)$ *such that* $(s^o, d^o) = (0, r)$ *is socially optimal if and only if* $\underline{k} \leq k^*$*.*

It is worth noting that the result of Proposition 6 does not imply that secured debt could not add value if $\underline{k} \leq k^*$. If the project had positive NPV if and only if some secured debt is used $(r - \gamma < 0)$, then it could be financed only if some secured debt is used. Specifically, if

$$\lambda r + (1-\lambda)(pk + \ell) - (1 - p\underline{k})\gamma - p\underline{k} > 0,$$

then the project can be financed provided that the bank issues enough secured debt.

Finally, we show that a higher collateral price p (similarly for k) reduces the probability of runs in the socially optimal funding arrangement.

4.2.2 Private Funding Choice

The bank's problem is to choose a funding structure (s, d) that maximize its payoff subject to the participation constraint:

$$\max_{s,\, d} V_B(s, d) \tag{21}$$

$$\text{subject to}$$

$$V_U(s, d) \geq \gamma,\ s \in [0, p\underline{k}] .$$

In choosing its optimal funding structure, the bank faces a tradeoff between the cost of financing and the expected deadweight loss. The cost of financing is decreasing in the face value of unsecured debt d. As the unsecured lenders' required payoff is greater than for secured lenders, increasing the proportion of secured debt reduces the average cost of financing. However, lower d makes runs more likely, which increases the expected deadweight loss.

Proposition 7 (Private Inefficiency with Repo) *The probability of bankruptcy under the socially optimal funding structure is always lower than under the bank's financing policy:* $\theta^*(s^o, d^o) < \theta^*(s^*, d^*)$.

In graphic terms, because the private choice of d is lower than for the social planner, it produces an upward shift of the $\theta^*(s^*, d)$ curve with a higher intercept at $s = 0$. The curve also exhibit increasing concavity. In conclusions, the private choice of s^* is either equal or higher than the social optimum value. Even when it is equal, it is combined with a lower value for d, as shareholders prefer to earn more in solvent states than reducing further the chance of runs. This leads to a higher threshold $\theta^*(s^*, d)$, and thus more frequent runs than the social optimum.

Proposition 8 characterizes the optimal private funding choice.

Proposition 8 (Private Funding with Repo) *The bank's financing policy is characterized as follows:*

(i) There exists a cutoff $\lambda_1 \in [0, 1)$ such that, if $\lambda \geq \lambda_1$, the bank's financing policy (s^*, d^*) has the bank borrowing either by issuing only unsecured debt ($s^* = 0$) or by issuing the maximum possible amount of secured debt ($s^* = p\underline{k}$). The optimal face value of unsecured debt d^* is characterized by either $\mu^* [V_U(s^*, d^*) - \gamma] = 0$ or $-\frac{\partial DW(s^*,d^*)}{\partial d} = \frac{\partial V_U(s^*,d^*)}{\partial d} [1 - s^* - \mu^*]$, where μ^* is the Lagrange multiplier associated with unsecured lenders' participation constraint.

(ii) There exists a cutoff $\lambda_2 \in (0, 1)$ such that, if $\lambda > \lambda_2$, unsecured lenders' participation constraint binds, i.e., $V_U(s^*, d^*) - \gamma = 0$.

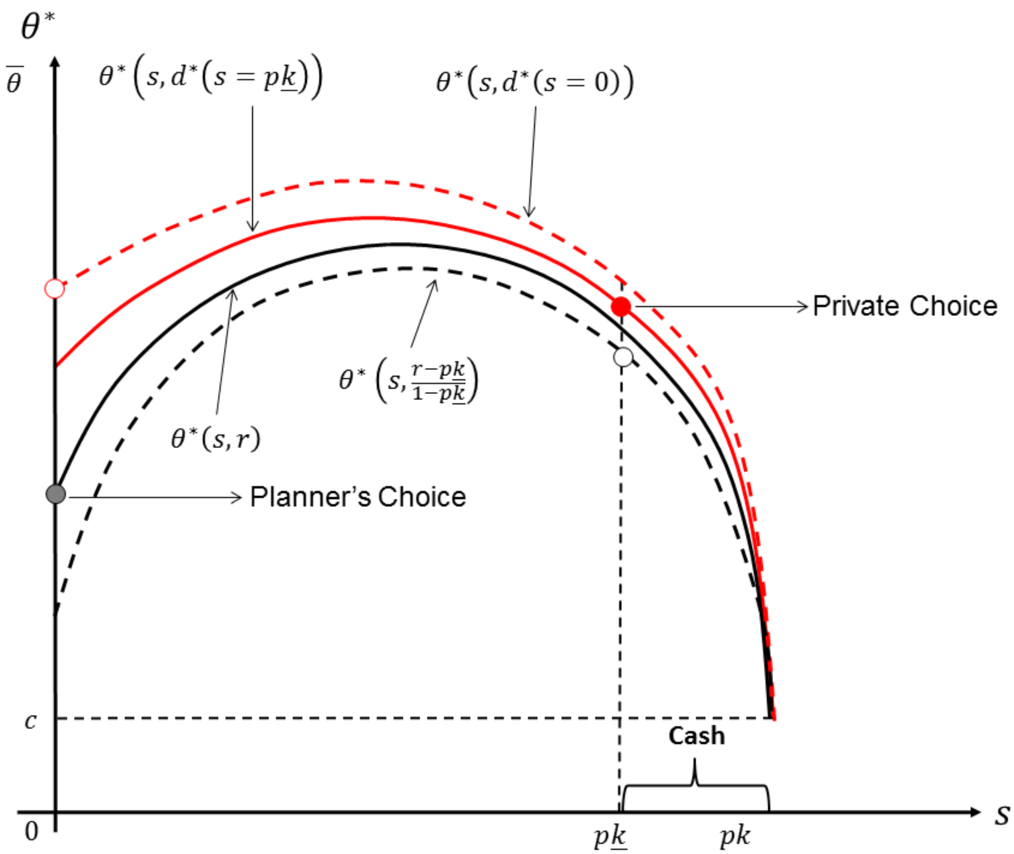
(iii) There exists a cutoff $\lambda_3 \in [0, 1)$ such that, if $\lambda > \max\{\lambda_1, \lambda_2, \lambda_3\}$, the bank borrows by issuing the maximum possible amount of secured debt ($s^* = p\underline{k}$). The optimal face value of unsecured debt d^* is characterized by unsecured lenders' breakeven condition $V_U(s^*, d^*) - \gamma = 0$.

The optimal funding structure is a corner solution because the bank's payoff is quasiconvex in s when λ is sufficiently high. The face value of unsecured debt balances the lower cost of funding against the higher expected deadweight loss from reducing d subject to the participation constraint. The condition for d^* is sufficient as we show that $V_B(s, d)$ is concave in d.

5 Deposit Insurance

In this section, we extend our model to include the possibility that a third party, such as a regulator, provides deposit insurance (DI) to unsecuder lenders. Consistent with real practice, we model DI as a minimum payment of $\pi \in [0, 1]$ for unsecured lenders in all states.

Figure 6: Private Funding Choice



In the presence of DI, the payoff of unsecured lenders who do not roll over in $t = 1$ is

$$\pi_U^N(\phi, \theta) = \begin{cases} 1, & \text{if } u(1-\phi) \leq \mathbf{1}_{\{\theta \geq c\}}(\theta - c) + p(k - sh) \\ \frac{1-\phi^*}{1-\phi} + \left(1 - \frac{1-\phi^*}{1-\phi}\right)\max\left\{\frac{\ell}{\phi^* u}, \pi\right\}, & \text{if } u(1-\phi) > \mathbf{1}_{\{\theta \geq c\}}(\theta - c) + p(k - sh) \end{cases},$$

while that of those who roll over is

$$\pi_U^R(\phi, \theta) = \begin{cases} \mathbf{1}_{\{\theta \geq c\}} d + \left(1 - \mathbf{1}_{\{\theta \geq c\}}\right)\max\{\rho, \pi\}, & \text{if } u(1-\phi) \leq \mathbf{1}_{\{\theta \geq c\}}(\theta - c) + p(k - sh) \\ \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}, & \text{if } u(1-\phi) > \mathbf{1}_{\{\theta \geq c\}}(\theta - c) + p(k - sh) \end{cases}. \tag{22}$$

Therefore, unsecured lender's net payoff of rolling over relative to that of running is

$$\Pi_U^R(\phi,\theta) = \begin{cases} \mathbf{1}_{\{\theta \geq c\}} d + \left(1 - \mathbf{1}_{\{\theta \geq c\}}\right) \max\{\rho, \pi\} - 1, & \text{if } u(1-\phi) \leq \mathbf{1}_{\{\theta \geq c\}}(\theta - c) + p(k - sh) \\ -\frac{pk-s}{(1-\phi)(1-s)}\left(1 - \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}\right), & \text{if } u(1-\phi) > \mathbf{1}_{\{\theta \geq c\}}(\theta - c) + p(k - sh) \end{cases}. \tag{23}$$

Similar to Diamond and Dybvig (1984), if the regulator provides full insurance, $\pi = 1$, then it is a dominant strategy to roll over regardless of the uncertain liquidation value of the assets θ and the fraction of unsecured lenders that roll over ϕ. That is, full insurance fully deters runs and achieves efficiency. If the amount of DI is such that $\pi \leq \min\left\{\frac{\ell}{1-pk}, \rho\right\}$, the payoffs are the same as those without the presence of DI and all the previous results go through.

We are thus left with the following two cases: $\min\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi \leq \max\left\{\frac{\ell}{1-pk}, \rho\right\}$ and $\max\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi < 1$. As before, the equilibrium cutoff θ_{DI}^* can then be computed by the threshold type who must be indifferent between rolling over and withdrawing given his beliefs about ϕ:

$$\int_0^{1-\frac{\theta_{DI}^* - c + pk - s}{1-s}} \left[-\frac{pk-s}{(1-\phi)(1-s)}\left(1 - \max\left\{\frac{\ell}{1-pk}, \pi\right\}\right)\right] d\phi + \int_{1-\frac{\theta_{DI}^* - c + pk - s}{1-s}}^1 (d-1)\, d\phi = 0. \tag{24}$$

This leads us to Proposition 9:

Proposition 9 (Run Cutoff with DI) *Suppose* $\min\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi < 1$. *In the limit* $\sigma \to 0$, *the unique equilibrium in* $t = 1$ *has unsecured lenders following monotone strategies with threshold* θ^* *given by*

$$\theta_{DI}^* = (1-s)\, e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1 - \max\left\{\frac{\ell}{1-pk}, \pi\right\}\right)}\right)} + c - (pk - s), \tag{25}$$

where all unsecured lenders roll over if $\theta > \theta^*$ *and do not roll over if* $\theta < \theta^*$.

The results in Corollary 5 below follow from Proposition 9.

Corollary 5 (DI Effect on Stability) *If* $\pi = 1$*, then there is no run in the presence of DI. If* $\pi \leq \min\left\{\frac{\ell}{1-pk}, \rho\right\}$*, the probability of bankruptcy with DI and without DI are the same:* $\theta^*_{DI} = \theta^*$*. If* $\min\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi < 1$*, the probability of bankruptcy with DI is at least as low as that without DI:* $\theta^*_{DI} = \theta^*$ *for* $\pi \leq \frac{\ell}{1-pk}$ *and* $\theta^*_{DI} < \theta^*$ *for* $\pi > \frac{\ell}{1-pk}$*, in which case* θ^*_{DI} *is strictly decreasing in* π*.*

The results above show that for any given private funding choice, an increase in the level of DI from π to $\pi' > \pi$ reduces the probability of bankruptcy (provided that π is sufficiently large). The natural question that arises is whether the same result holds taking into the dependence of the bank's funding choice on the level of DI.

If the high state is sufficiently likely (λ large enough), then Proposition 8 tells us that the bank issues the maximum possible amount of secured debt, $s^* = p\underline{k}$, and the face value of unsecured debt is determined by unsecured lenders' participation constraint $V_U\left(p\underline{k}, d^*; \pi\right) = \gamma$. An increase in π, *directly reduces* the probability of bankruptcy as $\theta^*\left(p\underline{k}, d^*; \pi'\right) < \theta^*\left(p\underline{k}, d^*; \pi\right)$, which increases unsecured lenders' expected payoff $V_U\left(p\underline{k}, d^*; \pi'\right) > V_U\left(p\underline{k}, d^*; \pi\right) = \gamma$. Thus, the bank's is able to reduce the face value of debt to $d^{*\prime} < d^*$ such that $V_U\left(p\underline{k}, d^{*\prime}; \pi'\right) = V_U\left(p\underline{k}, d^*; \pi\right) = \gamma$, which *indirectly increases* the probability of bankruptcy: $\theta^*\left(p\underline{k}, d^{*\prime}; \pi\right) > \theta^*\left(p\underline{k}, d^*; \pi\right)$. Corollary 6 below shows that the direct effect dominates when λ is sufficiently large.

Corollary 6 (DI Effect on Private Inefficiency) *Suppose* $\frac{\ell}{1-pk} < \pi < 1$ *and* λ *is sufficiently large. Then under the private funding choice with DI, both the face value of unsecured debt* d^* *and the probability of bankruptcy* $\theta^*\left(p\underline{k}, d^*\right)$ *are strictly decreasing in* π*.*

The intuition behind the result in Corollary 5 is simple. If the high state is sufficiently likely, then unsecured lenders' ex ante payoff is highly sensitive to the face value of usecured debt. Therefore, small drops in the face value d rapidly offset the gains brought about by decreases in probability of bankruptcy. As a result, the bank is unable to significantly reduce the face value of unsecured debt following an increase in the level of DI.

6 Conclusion

This paper examines run incentives under asset liquidity and fundamental risk. We obtain an unique run equilibrium thanks to a precise characterization of the bank default process. Existing run models assume that withdrawals are met by asset sales until none are left. In reality, less liquid assets cannot be sold immediately without huge losses. To avoid a hasty termination of real projects, bankruptcy law forces an automatic stay on all lenders once the borrower runs out of liquid assets. Remaining assets are then sold under orderly resolution, limiting fire sales of very illiquid assets. In this setup we are able to show that asset liquidity risk may cause runs even as fundamental risk vanishes.

The setup also enables to evaluate the effect of secured financial credit on run incentives. Introducing repo debt targeted to risk intolerant investors reduces the cost of funding. However, as secured credit claims the safest assets, it makes each unit of unsecured debt more exposed to risk, requiring a higher rollover yield. This shifts the signal threshold for asset liquidity, leading to more self protecting runs (Goldstein and Pauzner, 2005). In equilibrium, more secured debt results in more frequent unsecured runs. When illiquidity is rare, intermediaries seek inexpensive repo funding even if they recognize the increased risk of unsecured debt runs.

The direct effect of repo debt described here complements the indirect effect from collateral fire sales, which reduces collateral liquidity. As this forces more asset pledges to risk intolerant repo lenders, it reinforces the direct effect. In equilibrium, the reliance on secured funding is excessive relative to the social optimum from a stability perspective. Its cost may lower discount rates for marginal projects, and thus expand credit, just as lower interest rates do. This may have a procyclical effect on credit volume as well as on risk incentives.

A key question for future research concerns the effect of encumbered assets on stability when disclosure is limited. This reinforces market segmentation between traditional bank funding and its secured transactions, such as derivatives (Acharya and Bisin, 2013). Imprecise disclosure may create Knightian uncertainty and self fulfilling panics (Caballero Khrisnamurthy, 2008), even before private information on fundamental values becomes information sensitive

(Gorton and Ordoñez, 2014).

Appendix

Proof of Proposition 1. Goldstein and Pauzner (2000) and Morris and Shin (2003) prove this result for a general class of global games, including those where θ is drawn from a uniform distribution on $\left[\underline{\theta}, \overline{\theta}\right]$, the noise terms η_i are i.i.d. accross players and drawn from a uniform distribution on $\left[-\frac{1}{2}, \frac{1}{2}\right]$, and that satisfy the following additional conditions: (i) $\Pi_U^R(\phi, \theta)$ is nondecreasing in ϕ whenever $\Pi_U^R(\phi, \theta) > 0$; (ii) $\Pi_U^R(\phi, \theta)$ is nondecreasing in θ; (iii) there exists a unique θ^* that satisfies $\int_0^1 \Pi_U^R(\phi, \theta^*)\, d\phi = 0$; (iv) there exists $\overline{d}$ and $\underline{d}$ with $\sigma < \min\left\{\overline{\theta} - \overline{d}, \underline{d} - \underline{\theta}\right\}$, and $\epsilon > 0$ such that $\Pi_U^R(\phi, \theta) \leq -\epsilon$ for all $\phi \in [0,1]$ and $\theta \leq \underline{d}$ and $\Pi_U^R(\phi, \theta) > \epsilon$ for all $\phi \in [0,1]$ and $\theta \geq \overline{d}$; and (v) continuity of $\int_0^1 w(\phi)\, \Pi_U^R(\phi, \theta)\, d\phi$ with respect to signal θ and density w. Except for (iii), $\Pi_U^R(\phi, \theta)$ clearly satisfies (i), (ii), (iv) and (v).

We now show that (iii) is also satisfied. Let $\Delta(\theta; d) \equiv \int_0^1 \Pi_U^R(\phi, \theta)\, d\phi$. Since $\Delta(\theta; d) < 0$ for all d and $\theta < c$, then if θ^* exists it must be that $\theta^* \geq c$. Moreover, since $\Delta(\theta; d)$ is strictly increasing in θ for $\theta \geq c$, we must show that $\Delta(c; d) \leq 0$ for all d (otherwise for some d we have $\Delta(\theta; d) \geq \Delta(c; d) > 0$ for all $\theta \geq c$ and no θ^* would satisfy $\Delta(\theta^*; d) = 0$). It is straightforward to show that (a) $\Delta(c; d)$ is strictly increasing in d, (b) d is bounded by r (in which case the bank's participation constraint binds), and (c) that $\Delta(c; r) \leq 0$ if $e^{-\frac{r-1}{1-\frac{\ell}{1-pk}}} \geq pk$. Therefore, for $\frac{r-1}{1-\ell-pk} \leq 1$ we have

$$e^{-\frac{r-1}{1-\frac{\ell}{1-pk}}} > 1 - \frac{r-1}{1-\frac{\ell}{1-pk}} = (1-pk)\left(1 - \frac{r-1}{1-\ell-pk}\right) + pk \geq pk,$$

which implies that for all d we have $\Delta(c; d) \leq \Delta(c; r) \leq 0$. In addition, for all d we have $\Delta(\theta; d) > 0$ for θ sufficiently large such that there exists $\theta^* \geq c$ that satisfies $\Delta(\theta^*; d) = 0$. Finally, there is a unique such θ^* as $\Delta(\theta; d)$ is strictly increasing in θ for $\theta \geq c$.

For the derivation of the cutoff θ^*, note that after integrating the left-hand side of (14) we obtain

$$pk\left(1 - \frac{\ell}{1-pk}\right) \ln(\theta^* - c + pk) + (\theta^* - c + pk)(d-1) = 0. \tag{A.1}$$

After some algebra, (A.9) can be rewritten as

$$\frac{d-1}{pk\left(1 - \frac{\ell}{1-pk}\right)} = -\ln(\theta^* - c + pk)\, e^{-\ln(\theta^* - c + pk)}. \tag{A.2}$$

Let $W(\cdot)$ be the inverse function of $y = xe^x$ for $x \geq -1$ (known as the Lambert W function), that is, $x = W(y)$. Combined with (A.10) this implies

$$\theta^* = e^{-W\left(\frac{d-1}{pk\left(1-\frac{\ell}{1-pk}\right)}\right)} + c - pk,$$

which establishes the result. ■

Proof of Corollary 1. Implicitly differentiating $y = W(y)\, e^{W(y)}$ results in

$$W' = \frac{W}{(W+1)\, y} = \frac{e^{-W}}{1+W} > 0,$$
$$W'' = W'^2 \left(\frac{-2-W}{1+W}\right) < 0.$$

This allows us to compute

$$\frac{\partial \theta^*}{\partial d} = \frac{-e^{-W} W'}{pk\left(1 - \frac{\ell}{1-pk}\right)} < 0,$$
$$\frac{\partial^2 \theta^*}{\partial d^2} = \frac{e^{-W}\left(W'^2 - W''\right)}{\left[pk\left(1 - \frac{\ell}{1-pk}\right)\right]^2} > 0,$$
$$\frac{\partial \theta^*}{\partial (pk)} = -1 + \frac{W^2}{(d-1)(W+1)}\left[1 - \frac{\ell}{(1-pk)^2}\right].$$

If $(1-pk)^2 \leq \ell$, then $\frac{\partial \theta^*}{\partial (pk)} < 0$ and W is increasing in pk, which implies $\frac{\partial^2 \theta^*}{\partial (pk)^2} < 0$. If $(1-pk)^2 > \ell$, then $\frac{\partial \theta^*}{\partial (pk)}$ is positive for pk close enough to 0 and W is decreasing in pk, which implies that $\frac{\partial^2 \theta^*}{\partial (pk)^2} < 0$. Therefore, there exists $(pk)^* \in \left(0, 1-\sqrt{\ell}\right)$ such that $\frac{\partial \theta^*}{\partial (pk)} = 0$, with $\frac{\partial \theta^*}{\partial (pk)} > 0$ for $pk < (pk)^*$ and $\frac{\partial \theta^*}{\partial (pk)} < 0$ for $pk > (pk)^*$. ■

Proof of Corollary 2. This result follows from our assumptions that $pk > 0$ and that the return of project return is posisitvely associated with its risk ($r - 1 \leq 1 - \ell - pk$, which implies $pk < 1$), the concavity of θ^* relative to pk, and the constraint that the bank can promise at most r to unsecured lenders. Formally, we have that for all c

$$\begin{aligned}
\theta^* &\geq e^{-W\left(\frac{r-1}{pk\left(1-\frac{\ell}{1-pk}\right)}\right)} + c - pk \\
&= e^{-W\left(\frac{r-1}{1-\ell-pk}\frac{1-pk}{pk}\right)} + c - pk \\
&\geq e^{-W\left(\frac{1-pk}{pk}\right)} + c - pk \\
&> c\,.
\end{aligned}$$

■

Proof of Proposition 2. We show that the bank's participation constraint must bind at a solution d^o, in which case $d^o = r$. Suppose not, that is, $V_B\left(d^o\right) > 0$. The aggregate payoff $r - DW\left(d\right)$ is clearly increasing in d. The bank's payoff is strictly concave in d as

$$\overline{\theta}\frac{\partial V_B^2\left(d\right)}{\partial d^2} = 2\left(1-\lambda\right)\frac{\partial\theta^*}{\partial d} - \left(1-\lambda\right)\left(r-d\right)\frac{\partial^2\theta^*}{\partial d^2} < 0,$$

which in turn implies $V_B\left(d\right)$ is either (1) decreasing or (2) increasing and then decreasing since

$$\overline{\theta}\frac{\partial V_B\left(d\right)}{\partial d} = -\left[\overline{\theta} - \left(1-\lambda\right)\theta^*\right] - \left(1-\lambda\right)\left(r-d\right)\frac{\partial\theta^*}{\partial d} - \left(\overline{\theta}-1\right)$$

is negative for $d = r$. If $\frac{\partial V_B(d)}{\partial d} \leq 0$ for all d, then $V_B\left(d\right)$ is monotone decreasing. If $\frac{\partial V_B(d)}{\partial d} > 0$ for some d', then there exists d'' such that $\frac{\partial V_B(d)}{\partial d} = 0$. Since $V_B\left(d\right)$ is strictly concave in d, $\frac{\partial V_B(d)}{\partial d} > 0$ for $d < d''$ and $\frac{\partial V_B(s,d)}{\partial d} < 0$ for $d > d''$. Moreover, the bank's participation constraint binds when $d = r$. Therefore, the social planner can increase d^o until $V_B\left(d^o\right)$ binds: this increases the aggregate payoff while still satifying the constraints, which contradicts d^o being a solution. ■

Proof of Proposition 3. Suppose that $\theta^o\left(d^o\right) \geq \theta^*\left(d^*\right)$. Since we assume that r is sufficiently large, the bank's payoff under (21) is greater than zero. But then a contrat with d marginally greater than d^* satifies both participation constraints in (20) and results in $\theta^o\left(d^o\right) \geq \theta^*\left(d^*\right) > \theta^*\left(d\right)$. But this contradicts d^o being a solution to (20). ■

Proof of Proposition 4. The first order necessary conditions (FOC) are

$$-\frac{\partial DW(d)}{\partial d} = \frac{\partial V_U(d)}{\partial d}(1-\mu), \tag{A.3}$$

$$\mu[V_U(d) - \gamma] = 0, \tag{A.4}$$

$$V_U(d) \geq \gamma, \tag{A.5}$$

$$\mu \geq 0. \tag{A.6}$$

Since $V_B(d)$ is strictly concave (see Proof of Proposition 2), any d satisfying the FOC is a global maximizer, which shows (i).

For (ii), note that

$$\frac{\partial DW(d)}{\partial d} = \frac{(1-\lambda)}{\bar{\theta}}\frac{\partial \theta^*(d)}{\partial d}(r - pk - \ell), \tag{A.7}$$

$$\frac{\partial V_U(d)}{\partial d} = 1 - \frac{(1-\lambda)}{\bar{\theta}}\left[\theta^*(d) + \frac{\partial \theta^*(d)}{\partial d}(d - pk - \ell)\right]. \tag{A.8}$$

Consider $\mu = 0$. As λ gets close to 1, the left- and right-hand sides of (A.12) approach 0 ((A.16) approximates 0) and 1 ((A.17) converges to 1), respectively. Therefore, there are only two possibilities: either the left-hand side of (A.12) (strictly decreasing in λ) is smaller than the right-hand side (strictly increasing in λ) for all $\lambda \geq \lambda_1 = 0$, or there exists $\lambda(d) \in (0,1)$ such that the left-hand side of (A.12) is smaller than the right-hand side if $\lambda > \lambda(d)$ and at least as great if otherwise. If the former is true for all d, then (A.12) can only be satified if $\mu > 0$. Suppose there exists d such that the latter is true and denote Y the set of all such d. If $\lambda > \lambda_1 = \sup\{\lambda(d) : d \in Y\}$, then (A.12) can only be satified if $\mu > 0$. Combining these two possibilities we deduct that there exists a cutoff $\lambda_1 \in [0,1)$ such $\mu > 0$ if $\lambda > \lambda_1$, which in turn implies that $V_U(d) - \gamma = 0$ (from (A.13)). ■

Proof of Proposition 5. As in the proof of Proposition 1, it suffices to show that (iii) is also satisfied. Let $\Delta(\theta; s, d) \equiv \int_0^1 \Pi_U^R(\phi, \theta)\, d\phi$. Since $\Delta(\theta; s, d) < 0$ for all (s, d) and $\theta < c$, then if θ^* exists it must be that $\theta^* \geq c$. Moreover, since $\Delta(\theta; s, d)$ is strictly increasing in θ for $\theta \geq c$, we must show that $\Delta(c; s, d) \leq 0$ for all (s, d) (otherwise for some (s, d) we have $\Delta(\theta; s, d) \geq \Delta(c; s, d) > 0$ for all $\theta \geq c$ and no θ^* would satisfy $\Delta(\theta^*; s, d) = 0$). It is straightforward to show that (a) $\Delta(c; s, d)$ is strictly increasing in d, (b) d is bounded by $\frac{r-s}{1-s}$ (in which case the bank's participation constraint binds), (c) $\Delta\left(c; s, \frac{r-s}{1-s}\right)$ is decreasing in s if

$\frac{r-1}{1-\ell-pk} \leq \frac{1}{pk}$, and (d) that $\Delta(c;0,r) \leq 0$ if $e^{-\frac{r-1}{1-\frac{\ell}{1-pk}}} \geq pk$. Therefore, for $\frac{r-1}{1-\ell-pk} \leq 1$ we have

$$e^{-\frac{r-1}{1-\frac{\ell}{1-pk}}} > 1 - \frac{r-1}{1-\frac{\ell}{1-pk}} = (1-pk)\left(1 - \frac{r-1}{1-\ell-pk}\right) + pk \geq pk,$$

which implies that for all (s,d) we have $\Delta(c;s,d) \leq \Delta\left(c;s,\frac{r-s}{1-s}\right) \leq \Delta(c;0,r) \leq 0$. In addition, for all (s,d) we have $\Delta(\theta;s,d) > 0$ for θ sufficiently large such that there exists $\theta^* \geq c$ that satisfies $\Delta(\theta^*;s,d) = 0$. Finally, there is a unique such θ^* as $\Delta(\theta;s,d)$ is strictly increasing in θ for $\theta \geq c$.

For the derivation of the cutoff θ^*, note that after integrating the left-hand side of (14) we obtain

$$\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)\ln\frac{\theta^*-c+pk-s}{1-s} + \frac{\theta^*-c+pk-s}{1-s}(d-1) = 0. \tag{A.9}$$

After some algebra, (A.9) can be rewritten as

$$\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)} = -\ln\frac{\theta^*-c+pk-s}{1-s}e^{-\ln\frac{\theta^*-c+pk-s}{1-s}}. \tag{A.10}$$

Let $W(\cdot)$ be the inverse function of $y = xe^x$ for $x \geq -1$ (known as the Lambert W function), that is, $x = W(y)$. Combined with (A.10) this implies

$$\theta^* = (1-s)\,e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\right)} + c - (pk-s).$$

■

Proof of Corollary 3. Differentiating θ^* with respect to s allows us to compute

$$\frac{\partial\theta^*}{\partial s} = 1 - \frac{1-\frac{\ell}{1-pk}}{d-1}W\left(1-\frac{1}{W+1}\frac{1-pk}{1-s}\right),$$
$$\frac{\partial^2\theta^*}{\partial s^2} = W''\frac{(1-pk)^2}{(pk-s)^3}\frac{d-1}{\left(1-\frac{\ell}{1-pk}\right)} < 0,$$

Since $\lim_{s\to pk}\frac{\partial\theta^*(s,d)}{\partial s} = -\infty$ and $\theta^*(s,d)$ is strictly concave in s, it follows that $\theta^*(s,d)$ is strictly decreasing in s if $\frac{\partial\theta^*(0,d)}{\partial s} \leq 0$, and first strictly increasing, then strictly decreasing in s

if $\frac{\partial\theta^*(0,d)}{\partial s} > 0$. Note that

$$\begin{aligned}
\lim_{d\to1}\frac{\partial\theta^*(s,d)}{\partial s} &= 1-\left(1-\frac{\ell}{1-pk}\right)\left(1-\frac{1-pk}{1-s}\right)\lim_{d\to1}\frac{W}{d-1} \\
&= 1-\left(1-\frac{\ell}{1-pk}\right)\left(1-\frac{1-pk}{1-s}\right)\lim_{d\to1}\frac{W'\frac{1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}}{1} \\
&= 1-\left(\frac{pk-s}{pk-s}\right) = 0.
\end{aligned}$$

Because the above limit is true for any s, we have that $\frac{\partial\theta^*(0,d)}{\partial s}$ goes to zero as $d\to1$. We also have that

$$\frac{\partial^2\theta^*(s,d)}{\partial s\partial d} = \frac{1-\frac{\ell}{1-pk}}{(d-1)^2}W^2\left[\frac{1}{W+1}-\frac{1-pk}{1-s}\frac{W+2}{(W+1)^3}\right], \tag{A.11}$$

whose sign is determined by the the term inside the brackets.

If $pk=\frac{1+s}{2}$, then $\frac{\partial^2\theta^*(s,d)}{\partial s\partial d}=0$ for $d=1$ and $\frac{\partial^2\theta^*(s,d)}{\partial s\partial d}>0$ for $d>1$. If $pk>\frac{1+s}{2}$, $\frac{\partial^2\theta^*(s,d)}{\partial s\partial d}$ is positive for all $d\geq1$. Thus, if $pk\geq\frac{1+s}{2}$, it follows that $\frac{\partial^2\theta^*(s,d)}{\partial s\partial d}>0$ for all $d>\underline{d}=1$. This, in turn, implies that $\frac{\partial\theta^*(0,d)}{\partial s}>0$ whenever $pk\geq\frac{1}{2}$.

If $pk<\frac{1+s}{2}$, then $\frac{\partial^2\theta^*(s,d)}{\partial s\partial d}=0$ when $W(s,d)=\frac{-(1+pk-2s)+\sqrt[2]{(pk)^2-pk(6-4s)+5-4s}}{2}$. The term inside brackets in (A.11) is strictly increasing in $W(s,d)$, which in turn is strictly increasing in d. Since $W(s,d)$ grows without bounds as d increases and $W(s,1)=0$, it follows that there exists $\underline{d}(s)>1$ such that $W(s,\underline{d}(s))=\frac{-(1+pk-2s)+\sqrt[2]{(pk)^2-pk(6-4s)+5-4s}}{2}$. Therefore, $\frac{\partial^2\theta^*(s,d)}{\partial s\partial d}>0$ for $d>\underline{d}(s)$ and $\frac{\partial^2\theta^*(s,d)}{\partial s\partial d}\leq0$ for $d\leq\underline{d}(s)$. This implies that $\frac{\partial\theta^*(0,d)}{\partial s}>0$ for $d>\underline{d}(0)$ and $\frac{\partial\theta^*(0,d)}{\partial s}\leq0$ for $d\leq\underline{d}(0)$. ■

Proof of Corollary 4. We have that

$$\begin{aligned}
\frac{\partial\theta^*(s,r)}{\partial(pk)} &= -(1-s)W'e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\right)}\frac{(d-1)\left[\ell(1-s)-(1-pk)^2\right]}{\left[(pk-s)(1-pk-\ell)\right]^2}-1 \\
&= -\frac{1}{(d-1)(1-s)}\frac{W^2}{W+1}\left[\frac{\ell(1-s)-(1-pk)^2}{(1-pk)^2}\right]-1,
\end{aligned}$$

If $(1-pk)^2\leq\ell(1-s)$, then $\frac{\partial\theta^*}{\partial(pk)}<0$ and W is increasing in pk, which implies $\frac{\partial^2\theta^*}{\partial(pk)^2}<0$. If $(1-pk)^2>\ell(1-s)$, then $\frac{\partial\theta^*}{\partial(pk)}$ is positive for pk close enough to s and W is decreasing in pk, which implies that $\frac{\partial^2\theta^*}{\partial(pk)^2}<0$. Therefore, there exists $(pk)^*\in\left(s,1-\sqrt{\ell(1-s)}\right)$ such that $\frac{\partial\theta^*}{\partial(pk)}=0$, with $\frac{\partial\theta^*}{\partial(pk)}>0$ for $pk<(pk)^*$ and $\frac{\partial\theta^*}{\partial(pk)}<0$ for $pk>(pk)^*$. Since $\frac{\partial^2\theta^*}{\partial(pk)\partial s}>0$ for $pk\in\left(s,1-\sqrt{\ell(1-s)}\right)$ and $\frac{\partial^2\theta^*}{\partial(pk)^2}<0$, we have that $(pk)^*$ is increasing in s.

In addition, suppose $(1-pk)^2 \leq \ell$. If $\frac{\ell-(1-pk)^2}{\ell} \geq s$, then $\frac{\partial\theta^*(s,r)}{\partial(pk)}$ is negative. If $\frac{\ell-(1-pk)^2}{\ell} < s$, then $\frac{\partial\theta^*(s,r)}{\partial(pk)}$ is positive for s close enough to pk, negative for s close enough to $\frac{\ell-(1-pk)^2}{\ell}$, and $\frac{\partial^2\theta^*}{\partial(pk)\partial s} > 0$. Therefore, there exists $s^* \in \left(\frac{\ell-(1-pk)^2}{\ell}, pk\right)$ such that $\frac{\partial\theta^*(s,r)}{\partial(pk)} > 0$ for $s > s^*$ and $\frac{\partial\theta^*(s,r)}{\partial(pk)} < 0$ for $s < s^*$. That is, increasing the amount of safe assets reduces runs in the absence of secured debt ($s = 0$) and might increase runs in the presence of secured debt ($s > 0$). ∎

Proof of Proposition 6. We first show that the bank's participation constraint must bind at a solution (s^o, d^o). Suppose not, that is, $V_B(s^o, d^o) > 0$. The aggregate payoff $r - DW(s, d)$ is increasing in d, while the bank's payoff is either one of the following: (1) decreasing, or (2) increasing and then decreasing. The latter follows from the fact that

$$\overline{\theta}\frac{\partial V_B(s,d)}{\partial d} = -(1-s)\left[\overline{\theta} - (1-\lambda)\theta^*\right] - (1-\lambda)(r - d(1-s) - s)\frac{\partial\theta^*}{\partial d} - \left(\overline{\theta} - 1\right)(1-s)$$

is negative for $d = \frac{r-s}{1-s}$. If $\frac{\partial V_B(s,d)}{\partial d} \leq 0$ for all d, then $V_B(s,d)$ is monotone decreasing. If $\frac{\partial V_B(s,d)}{\partial d} > 0$ for some d', then there exists d'' such that $\frac{\partial V_B(s,d)}{\partial d} = 0$. Since $V_B(s,d)$ is strictly concave in d, $\frac{\partial V_B(s,d)}{\partial d} > 0$ for $d < d''$ and $\frac{\partial V_B(s,d)}{\partial d} < 0$ for $d > d''$. Moreover, the bank's participation constraint binds when $d = \frac{r-s}{1-s}$. Therefore, the social planner can increase d^o until $V_B(s^o, d^o)$ binds: this increases the aggregate payoff while still satifying the constraints, which contradicts (s^o, d^o) being a solution.

The result that the bank's participation constraint must be binding along with our assumption that r is sufficiently large assures that the unsecured lenders' participation constraint does not bind. The social planner's problem is therefore

$$\min_{s,\, d} \theta^*(s,d)$$

subject to

$$r - s - d(1-s) = 0,\ s \geq 0,\ s \leq p\underline{k}.$$

The first order necessary conditions (FOC) are

$$
\begin{aligned}
\frac{\partial \theta^*}{\partial s} - \lambda\left(d-1\right) - \mu_1 + \mu_2 &= 0, \\
\frac{\partial \theta^*}{\partial d} + \lambda\left(1-s\right) &= 0, \\
r - s - d\left(1-s\right) &= 0, \\
\mu_1 s &= 0, \\
\mu_2\left[p\underline{k} - s\right] &= 0, \\
\mu_1, \mu_2 &\geq 0.
\end{aligned}
$$

We now show that an interior optimum does not exist. Suppose not, i.e., there exists an interior optimum $\left(s^o, d^o\right)$. In this case, $\mu_1 = \mu_2 = 0$ and it must be that $-\frac{\frac{\partial \theta^*\left(s^o,d^o\right)}{\partial s}}{\frac{\partial \theta^*\left(s^o,d^o\right)}{\partial d}} = \frac{d-1}{1-s}$, which yields $\frac{W}{W+1}\frac{1-s}{pk-s} = e^W - 1$. This implies that a feasible decrease in s^* of Δs such that $\frac{\Delta d}{\Delta s} = \frac{d-1}{1-s}$ decreases $\theta^*\left(s^o, d^o\right)$:

$$
\begin{aligned}
&\frac{\partial \theta^*\left(s^o,d^o\right)}{\partial s}\Delta s + \frac{\partial \theta^*\left(s^o,d^o\right)}{\partial s}\Delta d \\
&= e^{-W}\left[e^W - 1 - \frac{1-pk}{pk-s}\frac{W}{W+1}\right]\Delta s - \frac{\left(1-s\right)e^{-W}W'}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\frac{d-1}{1-s}\Delta s \\
&= e^{-W}\left[e^W - 1 - \frac{1-s}{pk-s}\frac{W}{W+1} + \frac{W}{W+1}\right]\Delta s - e^{-W}\frac{W}{W+1}\Delta s \\
&= e^{-W}\left[e^W - 1 - \frac{1-s}{pk-s}\frac{W}{W+1}\right]\Delta s - e^{-W}\frac{W}{W+1}\Delta s + \frac{W}{W+1}\Delta s \\
&= \frac{W}{W+1}\left(1-e^{-W}\right)\Delta s < 0,
\end{aligned}
$$

which contradicts $\left(s^o, d^o\right)$ being a solution.

The previous result shows that $\theta^*\left(0, r\right) < \theta^*\left(s', \frac{r-s'}{1-s'}\right)$ for any given interior candidate $s' \in \left(0, p\underline{k}\right)$, which implies that that an optimum has either $s = 0$ or $s = p\underline{k}$. From the proof of Proposition 1, we know that $\theta^*\left(0, r\right) > c$. Since $\lim_{\underline{k}\to k}\theta^*\left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right) = c$, there exists a $k^* \in \left(\frac{s'}{p}, k\right)$ such that $\theta^*\left(0, r\right) \leq \theta^*\left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$ for $\underline{k} \leq k^*$ and $\theta^*\left(0, r\right) > \theta^*\left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$ for $\underline{k} > k^*$. ■

Proof of Proposition 7. Suppose that $\theta^o\left(s^o, d^o\right) \geq \theta^*\left(s^*, d^*\right)$. Since we assume that r is sufficiently large, the bank's payoff under (21) is greater than zero (the bank can garantee a positive payoff by choosing $s=0$). But then a contrat (s^*, d) with d marginally greater than d^* satifies both participation constraints in (20) and results in $\theta^o(s^o, d^o) \geq \theta^*(s^*, d^*) > \theta^*(s^*, d)$. But this contradicts (s^o, d^o) being a solution to (20). ■

Proof of Proposition 8. We first show (i). We use the Principle of Iterated Suprema to break the bank's problem into two stages, that is, we solve $\max_{d\in D}\left[\max_{s\in S} V_B(s,d)\right]$, where $S=[0, p\underline{k}]$ and $D=\{d : V_U(s^*(d), d) \geq \gamma\}$.

The next step is to show that $V_B(s,d)$ is quasiconvex in s if λ sufficiently high, which implies that there is not interior solution to problem $\max_{s\in S} V_B(s,d)$. This is done by showing that $\frac{\partial V_B(s,d)}{\partial s} = V_U(s,d) - 1 - (1-s)\frac{\partial V_U(s,d)}{\partial s} - \frac{\partial DW(s,d)}{\partial s}$ is a single crossing function, which is equivalent to $V_U - 1 - (1-s)\frac{\partial V_U}{\partial s}$ and $-\frac{\partial DW}{\partial s}$ satisfying signed-ratio monotonicity (Qua and Strulovici, 2012). Two functions $f(s)$ and $g(s)$ satisfy signed-ratio monotonicity if whenever $f(s) > 0$ and $g(s) < 0$, $-\frac{g(s)}{f(s)}$ is decreasing and whenever $f(s) < 0$ and $g(s) > 0$, $-\frac{f(s)}{g(s)}$ is decreasing. We take $g(s) = -\frac{\partial DW(s,d)}{\partial s}$ and $f(s) = V_U(s,d) - 1 - (1-s)\frac{\partial V_U(s,d)}{\partial s}$. Since $f(s)$ is always positive, we only need to consider the case in which $g(s) < 0$. In this case, it must be that $-\frac{g(s)' f(s) - g(s) f(s)'}{f(s)^2} < 0$. We have

$$\begin{aligned}
&\overline{\theta}\left[-g(s)' f(s) + g(s) f(s)'\right] = \\
&(1-\lambda)\theta^{*\prime\prime}(r - pk - \ell)\left\{(d-1)\left[\overline{\theta} - (1-\lambda)\theta^*\right] + (1-\lambda)\theta^{*\prime}\left[d(1-s) - (pk - s + \ell)\right]\right\} \\
&-(1-\lambda)\theta^{*\prime}(r - pk - \ell)\left\{(1-\lambda)\theta^{*\prime\prime}\left[d(1-s) - (pk - s + \ell)\right] - 2(1-\lambda)\theta^{*\prime}(d-1)\right\} \\
&= (1-\lambda)(r - pk - \ell)(d-1)\left[\theta^{*\prime\prime}\left(\overline{\theta} - (1-\lambda)\theta^*\right) + 2(1-\lambda)\theta^{*\prime}\theta^{*\prime}\right].
\end{aligned}$$

The sign of the above expression is determined by the term inside brackets, which is strictly decreasing in λ. For any given s, it is negative if λ is sufficiently close to 1, so that we are left with two possibilities: either it is nonpositive for all λ, or there exists $\lambda(s) \in (0,1)$ such that it is nonpositive if $\lambda \geq \lambda(s)$ and positive if otherwise. If the former is true for all s, then $V_B(s,d)$ is quasiconvex if $\lambda \geq \lambda_1 = 0$. Suppose there exists s such that the latter is true and denote X the set of all such s. Then $V_B(s,d)$ is quasiconvex if $\lambda \geq \lambda_1 = \sup\{\lambda(s) : s \in X\}$. Combining both cases we have that there exists a cutff $\lambda_1 \in [0,1)$ such that $V_B(s,d)$ is quasiconvex if $\lambda \geq \lambda_1$, which in turn implies that we must have a corner solution: $s^* \in \{0, p\underline{k}\}$.

We now turn to the problem $\max_{d \in D} V_B(s^*, d)$. The first order necessary conditions (FOC) are

$$-\frac{\partial DW(s^*, d)}{\partial d} = \frac{\partial V_U(s^*, d)}{\partial d}[1 - s^* - \mu], \tag{A.12}$$

$$\mu[V_U(s^*, d) - \gamma] = 0, \tag{A.13}$$

$$V_U(s^*, d) \geq \gamma, \tag{A.14}$$

$$\mu \geq 0. \tag{A.15}$$

To conclude the proof of (i) we need to show that any d satisfying the FOC is a global maximizer. This follows from

$$\overline{\theta}\frac{\partial V_B^2(s, d)}{\partial d^2} = 2(1 - s)(1 - \lambda)\frac{\partial \theta^*}{\partial d} - (1 - \lambda)(r - d(1 - s) - s)\frac{\partial^2 \theta^*}{\partial d^2} < 0,$$

which implies that $V_B(s^*, d)$ is (strictly) concave in d.

We now show (ii). Note that

$$\frac{\partial DW(s^*, d)}{\partial d} = \frac{(1 - \lambda)}{\overline{\theta}}\frac{\partial \theta^*(s^*, d)}{\partial d}(r - pk - \ell), \tag{A.16}$$

$$\frac{\partial V_U(s^*, d)}{\partial d} = 1 - \frac{(1 - \lambda)}{\overline{\theta}}\left[\theta^*(s^*, d) + \frac{\partial \theta^*(s^*, d)}{\partial d}\left(d - \frac{pk - s^* + \ell}{1 - s^*}\right)\right]. \tag{A.17}$$

Consider $\mu = 0$. As λ gets close to 1, the left- and right-hand sides of (A.12) approach 0 ((A.16) approximates 0) and $1 - s^*$ ((A.17) converges to 1), respectively. Since s is bounded above by $p\underline{k} < 1$, the right-hand side of (A.12) is bounded away from 0. Therefore, there are only two possibilities: either the left-hand side of (A.12) (strictly decreasing in λ) is smaller than the right-hand side (strictly increasing in λ) for all λ, or there exists $\lambda(s^*, d) \in (0, 1)$ such that the left-hand side of (A.12) is smaller than the right-hand side if $\lambda > \lambda(s^*, d)$ and at least as great if otherwise. If the former is true for all d, then (A.12) can only be satified if $\mu > 0$. Suppose there exists d such that the latter is true and denote Y the set of all such d. If $\lambda > \lambda_2 = \sup\{\lambda(s^*, d) : d \in Y\}$, then (A.12) can only be satified if $\mu > 0$. Combining these two possibilities we deduct that there exists a cutoff $\lambda_2 \in (0, 1)$ such $\mu > 0$ if $\lambda > \lambda_2$, which in turn implies that $V_U(s^*, d) - \gamma = 0$ (from (A.13)).

We finally show (iii). Suppose $\lambda > \max\{\lambda_1, \lambda_2\}$. In this case we know from (i) that there are two possible candidates for the bank's choice of secured debt: either $s^* = p\underline{k}$ or $s^* = 0$. We also know that unsecured lenders' participation constraint binds. Therefore, the bank's implied payoffs are given by

$$V_B\left(p\underline{k}, d^*\left(p\underline{k}\right)\right) = r - p\underline{k} - \left(1 - p\underline{k}\right)\gamma - DW\left(p\underline{k}, d^*\left(p\underline{k}\right)\right), \tag{A.18}$$

$$V_B\left(0, d^*\left(0\right)\right) = r - \gamma - DW\left(0, d^*\left(0\right)\right). \tag{A.19}$$

The difference is given by

$$V_B\left(p\underline{k}, d^*\left(p\underline{k}\right)\right) - V_B\left(0, d^*\left(0\right)\right) = p\underline{k}\left(\gamma - 1\right) - \left[DW\left(p\underline{k}, d^*\left(p\underline{k}\right)\right) - DW\left(0, d^*\left(0\right)\right)\right], \tag{A.20}$$

which is positive for λ sufficiently close to 1. Thus, there are two cases to consider: either (A.20) (strictly increasing in λ) is nonnegative for all $\lambda \geq \lambda_3 = 0$, or there exists $\lambda_3 \in (0, 1)$ such that (A.20) is nonnegative if $\lambda > \lambda_3$, and negative if $\lambda < \lambda_3$. Therefore, we conclude that if $\lambda > \max\{\lambda_1, \lambda_2, \lambda_3\}$, then the bank's financing policy has the bank borrowing by issuing only secured debt ($s^* = p\underline{k}$) and d^* is such that $V_U\left(s^*, d\right) - \gamma = 0$. ■

Proof of Proposition 9. Proof is analogous to those of Propositions 1 and 5. ■

Proof of Corollary 5. See discussion in text. ■

Proof of Corollary 6. For λ large enough, unsecured lenders' participation constraint binds: $V_U\left(p\underline{k}, d^*; \pi\right) = \gamma$. Thus, the overall change in θ^* caused by an increase in π can be found by differentiating both sides with respect to π, whihc yields:

$$\frac{\partial \theta^*\left(d, \pi\right)}{\partial d} d' + \frac{\partial \theta^*\left(d, \pi\right)}{\partial \pi} = \frac{\frac{(1-\lambda)\theta^*(d,\pi)}{\overline{\theta}}\left[d' - \frac{1-pk}{1-p\underline{k}}\right] - d'}{\frac{(1-\lambda)}{\overline{\theta}}\left[-\left(d-1\right) - \frac{1-pk}{1-p\underline{k}}\left(1-\pi\right)\right]}.$$

Since the denumerator on the right-hand side is negative and $d' < 0$, the overall expression is negative for λ large enough. ■

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