The Welfare Effects of Bank Liquidity and Capital Requirements

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Abstract

The stringency of bank liquidity and capital requirements should depend on their social costs and benefits. This paper investigates the welfare effects of these regulations and provides a quantification of their welfare costs. The special role of banks as liquidity providers is embedded in an otherwise standard general equilibrium growth model. In the model, capital and liquidity regulation mitigate moral hazard on the part of banks due to deposit insurance, which, if unchecked, can lead to excessive risk taking by banks through credit or liquidity risk. However, these regulations are also costly because they reduce the ability of banks to create net liquidity and they can distort capital accumulation. For the liquidity requirement, the reason is that safe, liquid assets are necessarily in limited supply and may have competing uses. A key insight is that equilibrium asset returns reveal the strength of preferences for liquidity, and this yields two simple formulas that express the welfare cost of each requirement as a function of observable variables only. Using U.S. data, the welfare cost of a 10 percent liquidity requirement is found to be equivalent to a permanent loss in consumption of about 0.03%. The cost of a similarly-sized increase in the capital requirement is found to be about five times as large. At the same time, the financial stability benefits of capital requirements are also found to broader.

1 Introduction

The 2007-2009 financial crisis has spurred vital financial reforms in many countries, including a strengthening of bank capital requirements and the introduction of new liquidity requirements, as part of the Basel III package of reforms. Still, an important debate continues on the question of whether these requirements have been increased sufficiently or whether there is more to be done. While there is widespread agreement that capital requirements can help make banks safer and that liquidity stress exacerbated the crisis, the ongoing debate in large part reflects differing views about the existence and magnitude of costs to society from imposing these restrictions on banks.

With regard to liquidity requirements, such as the LCR and NSFR rules of Basel III, there is an emerging literature on the theoretical benefits, based on mitgating externalities associated with bank runs and fire sales.[1] However, much less is known about their potential social costs. But if there are only benefits to liquidity requirements, why not raise them to levels that would force deposit-taking institutions to be narrow banks, with deposits backed exclusively by safe, liquid assets, as some have advocated?[2] The harm from bank runs and fire sales would presumably be greatly reduced, if not eliminated. Clearly, to determine the optimal level of liquidity requirements, the question of their social cost must be addressed.

Similarly, several studies have examined the theoretical benefits of capital requirements, based on the moral hazard associated with implicit or explicit government guarantees, or on other externalities associated with bank failures. Indeed, such requirements are the cornerstone of modern bank regulation and are enormously important for the capital structure of banks, especially large ones. Nevertheless, there is still much debate about the existence, let alone the magnitude, of any social costs involved with imposing restrictions on the capital structure of banks. Again, if there are only benefits to capital requirements, why not raise them to 100 percent and require that all bank assets be financed with equity? As is true for liquidity regulation, the optimal level of capital requirements can only be determined if the question of their social cost is addressed quantitatively.

This paper argues that liquidity and capital regulations can each impose an important cost for a similar reason: they reduce the ability of banks to create net liquidity through the transformation of illiquid loans into liquid deposits. After all, capital requirements directly limit the fraction of bank loans that can be financed by issuing deposit-type liabilities. Liquidity requirements force banks to hold safe, liquid assets against deposits, limiting

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[1] See, for example, Diamond and Kashyap (2015) and Calomiris, Heider and Hoerova (2015).

[2] A classic reference is Friedman (1960). See Cochrane (2014) for a recent proposal.

their liquidity transformation by restricting the asset side of their balance sheet. This can impose a social cost because safe, liquid assets are necessarily in limited supply and may have competing uses (see, for example, Krishnamurthy and Vissing-Jorgensson (2012) and Greenwood, Hansen and Stein (2015)).

The main contributions of this paper are to build a framework to analyze the social costs and benefits of liquidity and capital requirements, to derive simple formulas for the magnitude of the costs and to combine those formulas with data in order to measure the welfare costs of such requirements. All this is accomplished in a setting in which these regulations provide financial stability benefits by limiting moral hazard on the part of banks, and I analyze the degree to which capital and liquidity regulation are complements or substitutes in fostering financial stability.

The framework, based on Van den Heuvel (2008), embeds the role of liquidity creating banks in an otherwise standard general equilibrium growth model. Due to their role in the provision of liquidity services, bank liabilities are special and, as a result, the Modigliani-Miller theorem fails to hold for banks: their capital structure is not irrelevant. The welfare costs of the capital and liquidity requirements depend crucially on the value of the banks' liquidity creation. For this reason, households' preferences for liquidity are modeled in a flexible way. A key insight from the analysis is that equilibrium asset returns reveal the strength of these preferences for liquidity and this allows us to quantify the welfare costs without imposing restrictive assumptions on preferences. Furthermore, the analysis shows how capital and liquidity requirements can affect capital accumulation and the size of the banking sector. The formulas for the welfare costs take these general equilibrium feedbacks into account.

The model also incorporates a rationale for the existence of both capital and liquidity regulation, based on a moral hazard problem created by deposit insurance. Moral hazard can lead to excessive risk taking, which banks can engage in by making excessively risky loans and/or by holding insufficient liquid assets to guard against liquidity shocks. As bank supervision can only detect excessive risk taking imperfectly, a capital requirement is helpful in limiting the moral hazard problem by ensuring that shareholders internalize potential losses, be it from lending or from liquidity stress. A liquidity requirement is also helpful, though the model suggests that it only ameliorates the second manifestation of the moral hazard problem, excessive liquidity risk. However, it addresses this problem more directly than capital regulation and can therefore still be socially desirable.

The main findings are as follows. The preference for liquidity implies that the pecuniary rates of return on liquid assets – bank deposits and Treasuries – are lower than the returns

on non-liquid assets – equity in the model. For banks, this departure from Modigliani-Miller can result in a binding capital requirement. The liquidity requirement binds if the convenience yield on Treasuries exceeds the convenience yield on bank deposits, net of the noninterest cost of servicing those deposits. Because of competition, banks pass on both the cheap deposit funding to borrowers in the form of a lower lending rate. However, if binding, both the capital requirement and the liquidity requirement limit the degree of this pass-through. Possible noninterest costs of financial intermediation can also increase the lending rate. If the net impact of these factors is such that bank loans are still relatively inexpensive, firms will borrow exclusively from banks. Otherwise, the equilibrium will be one of both bank and non-bank finance, and the size of the banking sector will be determined endogenously.

As a consequence, liquidity and capital regulation can each lead to migration of financial activity to non-bank intermediaries, such as shadow banks, or to disintermediation. For liquidity regulation, this outcome is more likely if the supply of high quality liquid assets is low relative to the demand for such assets, so that their convenience yield is high. Moreover, these regulations can alter not only the composition of the financial sector, but also the size of the economy, through their effect on firm investment.

Turning to normative results, both capital and liquidity regulations are, as noted, helpful in mitigating moral hazard and preventing financial crises in the model. While the financial stability benefits of the capital requirement are broader, the liquidity requirement is a useful complement to ensure prudent liquidity risk management. Indeed, the model suggests a simple division of labor: It socially optimal for the liquidity requirement to address liquidity risk and for the capital requirement to deal with credit risk. That said, if the regulator has imperfect information about the severity of potential liquidity stress or credit risk, then the division of labor is less clear-cut. In that case, it may be optimal for capital regulation to consider liquidity risk as well as credit risk.

These benefits are not a free lunch, however, as these regulations also entail social costs. If binding, each requirement reduces banks' ability to engage in liquidity transformation, which is a socially valuable activity. The model can be used as a lens to see how the magnitude of these costs can be measured with real-world data. Specifically, equilibrium asset returns reveal the strength of investors' preferences for liquidity, which allows for the derivation of two simple formulas that express the marginal welfare costs of the regulations as functions of observable variables only. (The formulas are presented in section 3 of the paper, in propositions 5 and 6.)

The next section then uses U.S. data to measure these observable variables, most

crucially certain preference-revealing financial spreads. For liquidity regulation, the key spread is the one between bank deposits and Treasuries. For capital regulation it is the equity-deposit spread, although both spreads must be adjusted for banks' net noninterest cost of servicing deposits. The welfare cost of a 10 percent liquidity requirement is found to be equivalent to a permanent loss in consumption of about 0.03%, a modest cost. Using a conservative method, the cost of a similarly-sized increase in the capital requirement is found to be about five times as large.

As a caveat, the model does not feature a lender of last resort that could save solvent banks with liquidity problems, which could lessen the need for ex-ante liquidity regulation. Because of that, the analysis may overstate the beneficial role of liquidity regulation, tough it does not matter for the results on the cost. That said, in reality, the lender of last resort function of central banks is not completely free of challenges. Deciding whether a bank only experiences liquidity problems or liquidity and solvency problems could be difficult in crisis times. And it has been argued that interventions by a lender of last resort could themselves lead to moral hazard problems (see, e.g., Farhi and Tirole (2012)). To the extent that the lender of last resort function entails economic costs, these could be compared to the costs of liquidity regulation, which this paper attempts to quantify.[3]

There are several recent papers that present quantitative, macroeconomic models of optimal bank capital regulation, including Begenau (2015), Clerc at al. (2015) and Nguyen (2013). In their calibrated versions, these models each yield an interior level of the capital requirement that maximizes a welfare criterion, with the levels ranging from 8 percent in Nguyen, whose model features endogenous growth, to 14 percent in Begenau, which is more similar to a standard growth model. There are three main differences with the model presented below. First and most obviously, these papers do not aim to examine liquidity requirements, which is a focus in this paper. Second, the reason the Modigliani-Miller theorem fails for banks is different, except for Begenau, in which, as in this paper, it fails in part because banks provide liquidity services. Incidentally, this is also the key friction in Gorton and Winton (2014) and Van den Heuvel (2008), who also show that bank capital requirements may have an important social cost because they reduce the ability of banks to create liquidity. Third, Begenau , Clerc at al. and Nguyen all rely on a full calibration of the model to draw out quantitative implications, whereas the main results in this paper are obtained without calibration, using a revealed preference argument instead. This could

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[3]See Carlson, Duygan-Bump, and Nelson (2015) for a discussion of the relation between liquidity regulation and the lender of last resort. They are argue that each tool is needed in part to address the limitations of the other.

be viewed as attractive because macroeconomic models with financial intermediation have many parameters that can be notoriously difficult to calibrate. On the other hand, this paper only quantifies the welfare *cost* of regulation using this revealed-preference methodology, characterizing the benefits only qualitatively.

Finally, Diamond and Kashyap (2015) and Calomiris, Heider and Hoerova (2015) put forth a rationale for liquidity requirements in terms of preventing bank runs. De Nicolo, Gamba and Lucchetta (2014) conduct a quantitative examination of the effects of both liquidity and capital requirements. Adopting a microprudential perspective, they present a partial equilibrium model of banks that engage in maturity transformation and are tempted to take on excessive risk due to deposit insurance.

2   The Model

As mentioned, the model extends Van den Heuvel (2008). The key deviation of that model from the standard growth model is that households have a need for liquidity, and that certain institutions, labelled banks, are able to create financial assets, bank deposits, which provide liquidity services. As a novel feature in this model relative to its precursor, bonds issued by the government can also serve as liquid assets for households and businesses. In addition, government bonds can be used by banks to deal with liquidity risk and to satisfy liquidity regulation – the main other new features in this paper.

Since a central goal of the model is to provide a framework not just for illustrating, but for actually measuring the welfare cost of liquidity and capital requirements, it is important to model the preferences for liquidity in a way that is not too restrictive. As much as possible, the data should be allowed to provide the answer, not special modeling choices. To that end, I follow Sidrauski (1967) and a large literature in monetary economics in adopting the modeling device of putting liquidity services in the utility function.

The advantage of this approach is its flexibility. Crucially, all main results will be derived without making any assumptions on the functional form of the utility function, beyond the standard assumptions that it is increasing and concave, thus allowing the data to speak.

Of course, this approach does not further our understanding of why households like liquid assets, but this is simply not the topic of this paper. That said, it is important to know that the Sidrauski modeling device is functionally equivalent to a range of more specialized, micro-founded models of liquidity demand, such as the Baumol-Tobin transaction technology or cash-in-advance, as shown by Feenstra (1986). In that equivalence, the util-

ity function with money (or deposits) as an argument is simply a derived utility function. Because we will not impose any restrictions on that derived utility function, all results will hold for any of those more primitive models.[4]

The economy consists of households, banks, (nonfinancial) firms, and a government. Households own both the banks and the nonfinancial firms. These firms combine capital and labor to produce the single good. The rest of this section describes the environment, analyzes the agents' decision problems and concludes with a discussion of financial stability policy.

2.1 Households

There is a continuum of identical households with mass one. Households are infinitely lived dynasties and value consumption and liquidity services. Households can obtain these liquidity services by allocating some of their wealth to bank deposits, an asset created by banks for this purpose. In addition, government bonds also derive a convenience value from holding government bonds, which stems from their liquidity and safety.

Besides holding bank deposits, denoted d_t, households can store their wealth by holding equity, e_t, or government bonds, b_t. They supply a fixed quantity of labor, normalized to one, for a wage, w_t. Taxes are lump-sum and equal to T_t. There is no aggregate uncertainty, so the representative household's problem is one of perfect foresight:

$$\max_{\{c_t, d_t, b_t, e_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, d_t, b_t)$$
$$\text{s.t.} \quad d_{t+1} + b_{t+1} + e_{t+1} + c_t = w_t 1 + R_t^D d_t + R_t^B b_t + R_t^E e_t - T_t$$

and subject to a no-Ponzi-game condition and initial wealth constraint for $d_0 + b_0 + e_0$. c_t is consumption in period t, β is the subjective discount factor and R_t^D, R_t^B and R_t^E are the returns on bank deposits, government bonds, and (bank or firm) equity, respectively. The returns and the wage are determined competitively, so the household takes these as given. There is no distinction between bank and firm equity, since, in the absence of risk, they are perfect substitutes for the household and will thus yield the same return.

The utility function is assumed to be concave, at least once continuously differentiable on $\mathbb{R}_{++}^3$, increasing in all arguments, and strictly so in consumption: $u_c(c, d, b) \equiv \partial u(c, d, b)/\partial c > 0$, $u_d(c, d, b) \equiv \partial u(c, d, b)/\partial d \geq 0$ and $u_b(c, d, b) \equiv \partial u(c, d, b)/\partial b \geq 0$.

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[4]Maintaining this level of generality also avoids another potential disadvantage of the Sidrauski modeling device: If a particular functional form for the derived utility function is needed, that choice can be difficult to justify. For example, is the marginal utility of consumption increasing or decreasing in deposits? And to what degree, if any, are deposits and government bonds substitutes?

The first-order conditions to the household's problem are easily simplified to

$$R_t^E = (\beta u_c(c_t, d_t, b_t)/u_c(c_{t-1}, d_{t-1}, b_{t-1}))^{-1} \tag{1}$$

$$R_t^E - R_t^D = u_d(c_t, d_t, b_t)/u_c(c_t, d_t, b_t) \tag{2}$$

$$R_t^E - R_t^B = u_b(c_t, d_t, b_t)/u_c(c_t, d_t, b_t) \tag{3}$$

Equation (1), which determines the return on equity, is the standard intertemporal Euler equation for the consumption-saving choice, with one difference: the marginal utility of consumption may depend on deposits and bond holdings. Equation (2) states that the marginal utility of the liquidity services provided by deposits, expressed in units of consumption, should equal the spread between the return on equity and the return on bank deposits. This spread is the opportunity cost of holding deposits rather than equity. If $u_d > 0$, then the return on equity will be higher than the return on deposits to compensate for the fact that equity does not provide liquidity services. Equation (3) relates the spread between equity and bonds to the liquidity services of bonds in a similar fashion.

2.2 Banks

There is a continuum of banks with mass one, which make loans to nonfinancial firms, may hold government bonds and finance these assets by accepting deposits from households and issuing equity. The ability of banks to create liquidity through deposit contracts is their defining feature. Banks last for one period[5] and every period new banks are set up with free entry into banking, so banks operate in an environment of perfect competition. The balance sheet, and the notation, for the representative bank during period t is:

Assets		**Liabilities**	
L_t	Loans	D_t	Deposits
B_t	Bonds	E_t	Bank Equity

For quantitative realism the model allows for resource costs associated with servicing deposits and/or making loans. Specifically, a bank incurs a noninterest cost $g(D, L)$ to service those financial contracts.[6] g is assumed to be nonnegative, twice continuously differentiable, (weakly) increasing, convex and homogenous of degree 1, i.e. it exhibits constant returns to scale. Note that costless intermediation is included as a special case ($g \equiv 0$), as is a linear cost function.

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[5] This is without loss of generality, since there are no adjustment costs, nor any agency problems between banks and the other optimizing agents, households and firms.

[6] Note that the bank is assumed not to incur an intermediation cost from holding government bonds or from issuing or servicing equity.

Regulation   Banks are subject to regulation, as well as supervision, by the government. One form of regulation is deposit insurance. The deposit insurance fund ensures that no depositor suffers a loss in the event of a bank failure. That is, all deposits are fully insured. The rationale for the deposit insurance is left unmodeled. However, it has been argued that deposit insurance improves the ability of banks to create liquidity.[7]

Secondly, banks face a capital requirement, which requires them to have a minimum amount of equity as a fraction of risk-weighted assets. In the context of this simple model, the capital requirement states that equity needs be at least a fraction γ of loans for a bank to be able to operate:

$$E_t \geq \gamma L_t$$

For the moment, the capital requirement is merely assumed. It will later be shown how it can be socially desirable to have such a requirement, as it mitigates the moral hazard problem that arises due to the presence of deposit insurance. There is no rationale in the model for requiring equity against the bank's holdings of government bonds (which are assumed to be riskless). Accordingly, I have assumed that government bonds have a zero risk weight.

Thirdly, banks must satisfy a liquidity requirement by holding a minimum level government bonds, set equal to a fraction λ of deposits:

$$B_t \geq \lambda D_t$$

Again, for the moment this regulation is merely assumed, but later it will be shown how it can be socially desirable in the presence of liquidity risk and the externalities associated with deposit insurance.

Loans and credit risk   The bank can make safe or risky loans to nonfinancial firms. Riskless loans yield a gross rate of return R_t^L, for sure, at the end of the period t. R_t^L is determined competitively in equilibrium, so each bank takes it as given.

Deposit insurance creates a moral hazard problem: the bank has an incentive to engage in excessive risk taking. As this is the justification for the capital requirement, a way for the bank to add risk is introduced. Specifically, by directing a fraction of its lending to firms with a risky technology, described below, the bank can create a loan portfolio with riskiness σ_t that pays off $R_t^L + \sigma_t \varepsilon_t$, where ε_t is an idiosyncratic shock with negative mean, denoted $-\xi$ ($\xi \geq 0$). Further, it is assumed that ε has a cumulative distribution function,

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[7]Diamond and Dybvig (1983) provide a model of liquidity provision by banks, in which socially undesirable, panic based bank runs can occur, and in which deposit insurance can prevent these runs.

F_ε, that has bounded support $[\underline{\varepsilon}, \bar{\varepsilon}]$, with $-\infty < \underline{\varepsilon} < 0 < \bar{\varepsilon} < \infty$ and $\Pr[\varepsilon > 0] > 0$. ε is *i.i.d.* across banks and time periods.

The negative mean of the shock implies that the expected return of the loan portfolio is decreasing in its risk. It is in this sense that risk-taking is excessive: absent a moral hazard problem due to deposit insurance, the bank would always prefer $\sigma_t = 0$. While the bank chooses σ_t, bank supervision imposes an upper bound: $\sigma_t \leq \bar{\sigma}$. This will be explained more fully in the discussion of the government.

Deposits and liquidity risk Deposits come with liquidity risk for the bank. This feature is introduced not just for realism, but also to provide a rationale for liquidity regulation. Consistent with the design principles of the LCR, it is assumed that, with a small probability, an unusually high fraction of depositors decide to withdraw early, before the bank has received the income from the loans it has made. The occurrence of this event, termed "liquidity stress", can be thought of as the realization of bank-specific liquidity risk.[8]

The bank can cover these early withdrawals by drawing down its stock of liquid securities, i.e. its holdings of government debt. In contrast, loans are fully illiquid and no secondary market exists for loans. As a consequence, if the bank does not have sufficient government bonds to cover the intra-period withdrawals, the bank defaults and goes into bankruptcy protection. Shareholders get zero in this case, while depositors are made whole by the deposit insurance scheme. The resolution through the deposit insurance fund is discussed in more detail below. The assumption of complete illiquidity of loans is admittedly an extreme one. The key idea, however, is that loans are less liquid, especially in times of stress, and that this can make it desirable for banks to hold more liquid securities in anticipation of stressed outflows. The question then becomes whether the private incentives to hold liquid assets are as strong as the social benefits.

Formally, let η be a random variable that takes on the value one when liquidity stress materializes, and zero otherwise, and let p be the probability that a bank does *not* suffer liquidity stress. Thus, $\eta = 0$ with probability p and $\eta = 1$ with probability $1 - p$. It is assumed that η and ε are mutually independent, and, like ε, η is *i.i.d.* across banks and time periods. Denote the fraction of depositors who decide to withdraw early when $\eta = 1$ by w. Thus, a bank fails due to liquidity stress if $B < wD$ and $\eta = 1$. It is assumed that early withdrawers use their funds to make payments to other households, who then deposit the funds into the banking system. To economize on notation and avoid having to

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[8] The model is silent about whether this liquidity stress is panic-based or based on a fundamental (preference) shock.

keep track of intra-period balance sheet changes, I adopt the simplifying assumption that those banks that experienced the liquidity outflows are also shortly thereafter recipients of liquidity inflows of the same magnitude (regardless of whether they survived the acute liquidity stress or are in FDIC resolution). Although this is clearly not the most realistic assumption, it simplifies the analysis and more realistic assumptions would lengthen the exposition without yielding additional insights.

Finally, it is important to keep in mind that the assumptions regarding the deposit insurance, excessive risk taking through lending, liquidity stress and bank supervision matter only for the benefits of the capital and liquidity requirements, not for their welfare costs, nor for the measurement of these costs.

The bank's maximization problem   The objective of the bank is to maximize shareholder value, net of the initial equity investment:[9]

$$\pi^B = \max_{\sigma, L, B, D, E} \mathbb{E}\left[(1 - 1_{\{B < wD\}}\eta)\{(R^L + \sigma\varepsilon)L + R^B B - R^D D - g(D, L)\}^+\right]/R^E - E$$

$$\text{s.t. } L + B = E + D, \quad E \geq \gamma L, \quad B \geq \lambda D, \quad \text{and } \sigma \in [0, \bar{\sigma}] \tag{4}$$

The notation $\{x\}^+$ stands for $\max(x, 0)$ and $1_{\{B < wD\}}$ is an indicator variable taking the value 1 if $B < wD$ and zero otherwise, reflecting the fact that the bank will face bankruptcy due to liquidity stress if both $B < wD$ and $\eta = 1$. The constraints are, respectively, the balance sheet identity, the capital requirement, the liquidity requirement, and the supervisory bound on σ.

The term $(R^L + \sigma\varepsilon)L + R^B B - R^D D - g(D, L)$ is the bank's net cash flow at the end of the period, provided there was no failure due to liquidity stress. It consists of interest income from loans and bonds, minus any possible charge-offs on the loans, minus the interest owed to depositors, and minus the resource cost of intermediation. If the net cash-flow is positive, shareholders are paid this full amount in dividends. If the net cash flow is negative, the bank fails and the deposit insurance fund must cover the difference in order to indemnify depositors, as limited liability of shareholders rules out negative dividends. Shareholders receive zero in this event or if the bank has already failed due to liquidity stress, so dividends equal $(1 - 1_{\{B < wD\}}\eta)\{(R^L + \sigma\varepsilon)L + R^B B - R^D D - g(D, L)\}^+$. E is the initial investment of the shareholders. At the beginning of period t shareholders discount the value of end-of-period dividends by the opportunity cost of holding this particular bank's equity. This

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[9]In what follows, time subscripts will be used only where necessary to avoid confusion.

opportunity cost is R^E, the market return on equity. If $\sigma > 0$ or if $B < wD$, dividends are risky, but this risk is perfectly diversifiable, so shareholders do not price it.[10]

2.2.1 Analysis of the bank's problem

The solution to the bank's problem in (4) differs depending on whether or not the liquidity requirement exceeds stressed withdrawals. If $\lambda \geq w$, then regulation forces the bank to self-insure against the liquidity stress, simplifying the problem by effectively eliminating this risk. On the other hand, if $\lambda < w$, then the bank must decide whether or not to self-insure against liquidity problems. I start by analyzing the simpler case: $\lambda \geq w$. In that case, (4) simplifies as $1_{\{B<wD\}}\eta = 0$.

Choice of credit risk with $\lambda \geq w$ First, consider the choice of loan risk, σ, conditional on L, B, D and E. For convenience, define $r \equiv R^L + R^B(B/L) - R^D(D/L) - g(D/L, 1) > 0$, a measure of the bank's return on assets without excessive risk taking.[11] In this notation, expected dividends are $\mathbb{E}\left[\{(r + \sigma\varepsilon)L\}^+\right]$. Due to the max operator, this expectation is a convex function of σ.[12] For low values of σ, expected dividends are decreasing in σ, reflecting the negative mean of the shock ε – this is the cost of excessive risk-taking. But at higher levels of σ, there is not enough equity (which earns r) to absorb the loss in the event of a large negative realization of ε. In that event, the excess loss is covered by the deposit insurance fund. Increasing risk further at this point increases the payoff to shareholders in the good states ($\varepsilon > 0$) without lowering it in (all) bad states – this is the benefit of excessive risk-taking to shareholders. Put differently, the value of the put option associated with the deposit insurance fund increases with σ.[13]

Because of the convexity of expected dividends, only $\sigma = 0$ and $\sigma = \bar{\sigma}$ need to need to be considered as candidates for the optimal choice of risk. Comparing expected dividends for these two values yields the following result (see Appendix A.1 for details):[14]

$$\sigma = 0 \text{ if and only if } \phi_\varepsilon \bar{\sigma} \leq r \tag{5}$$

$$\sigma = \bar{\sigma} \text{ otherwise}$$

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[10] Hence, the treatment of R^E as nonstochastic in the household problem is also still correct, since, even if banks are risky, households would not leave any such risk undiversified.

[11] Recall that g is linear homogenous.

[12] See Appendix A.1 for proof of convexity.

[13] As an example, suppose ε equals either $-\xi + a$ or $-\xi - a$, with equal probability, and with $a > \xi$ (so that $\Pr[\varepsilon > 0] > 0$ as assumed). Then $\mathbb{E}\left[\{(r + \sigma\varepsilon)L\}^+\right] = \left\{ \begin{smallmatrix} (r - \sigma\xi)L \text{ if } r - \sigma(a+\xi) \geq 0 \\ 0.5(r + \sigma(a-\xi))L \text{ if } r - \sigma(a+\xi) \leq 0 \end{smallmatrix} \right.$

[14] This condition is identical to the one in Van den Heuvel (2008, equation 5), except that there $\phi_\varepsilon = 1$, reflecting a normalizing assumption adopted on the distribution of ε.

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where ϕ_ε is a "value-at-risk" statistic derived from the distribution of ε. It is implicitly defined by

$$\int_{\underline{\varepsilon}}^{-\phi_\varepsilon} (\phi_\varepsilon + \varepsilon) dF_\varepsilon(\varepsilon) \equiv -\xi \tag{6}$$

The assumptions made regarding the distribution function F_ε imply that ϕ_ε exists, is unique and satisfies $0 < \phi_\varepsilon \leq -\underline{\varepsilon}$ (see the appendix). For small values of ξ, $\phi_\varepsilon \approx -\underline{\varepsilon}$.[15] Intuitively, excessive risk taking is attractive if supervision is weak (high $\bar{\sigma}$), if the distribution of ε has a wide and fat left tail (high ϕ_ε), if the bank has little equity at stake (low r), or if the cost of excessive risk-taking is small (low ξ, which implies a higher value of ϕ_ε).[16]

Solution to the bank's problem with $\lambda \geq w$ and $\sigma = 0$ From now on, unless stated otherwise, it is assumed that the condition for $\sigma = 0$ shown in (5) holds.[17] With that and with $\lambda \geq w$, the bank's maximization problem in (4) simplifies to:

$$\pi^B = \max_{L,B,D,E} \mathbb{E}\left[R^L L + R^B B - R^D D - g(D,L)\right]/R^E - E \tag{7}$$

$$\text{s.t. } L + B = E + D, \quad E \geq \gamma L, \quad B \geq \lambda D$$

It is straightforward to solve this problem (see Appendix A.2). To summarize the results, it is convenient to first define the all-in cost of financing a unit of loans with deposits, taking into account the liquidity requirement (but setting aside sets aside the transaction costs $g(D,L)$):

$$\tilde{R}^D(\lambda) \equiv R^D + \frac{\lambda}{1-\lambda}(R^D - R^B) \tag{8}$$

This reflects the fact that a fraction λ of the deposits must be invested in bonds, rather than loans, so to finance one unit of loans with deposits, $1/(1-\lambda)$ deposits must be raised, of which $\lambda/(1-\lambda)$ are put in bonds. If the return on bonds is less than the interest paid to depositors, then the liquidity requirement effectively increases the cost of financing loans with deposits. For ease of exposition, I first characterize the results for the special case of zero resource costs of intermediation ($g \equiv 0$):[18]

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[15] As an example, for the illustrative distribution in the previous footnote, $\phi_\varepsilon = a - \xi$. As a further example based on a continuous distribution, suppose ε is uniformly distributed on the interval $[-\xi - a, -\xi + a]$ with $a > \xi$ (so that $\bar{\varepsilon} > 0$ as required). Then it is straightforward to show that $\phi_\varepsilon = \left(\sqrt{a} - \sqrt{\xi}\right)^2 \in (0, a)$.

[16] In more detail, ϕ_ε is the value at risk such that losses in excess of that value ($\varepsilon < -\phi_\varepsilon$) are in expectation just equal to the mean of ε. When $\phi_\varepsilon \sigma = r$, the bank becomes just insolvent when $\varepsilon = -\phi_\varepsilon$. In that situation, the expected benefits of risk shifting for the bank (the expected excess losses) are equal to the expected costs (the reduction in NPV due to the negative mean of ε).

[17] Since r depends on the choice variables L, B, D and E, I will verify later that the condition holds.

[18] This proposition is really a corollary to the more general proposition that follows it.

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Proposition 1 (Solution to the bank's problem when $g \equiv 0$, $\lambda \geq w$, and $\phi_\varepsilon \bar{\sigma} \leq r$)

For the special case of costless intermediation $(g \equiv 0)$, existence of a finite solution requires $R^B \leq R^D \leq R^L \leq R^E$. The solution satisfies the zero-profit condition:

$$R^L = \gamma R^E + (1-\gamma)\tilde{R}^D(\lambda) \tag{9}$$

resulting in $\pi^B = 0$. $\tilde{R}^D(\lambda) \geq R^D$, with strict inequality if and only if the liquidity requirement binds. Finally,

- *The liquidity requirement binds (so $B = \lambda D$) if and only if $R^B < R^D$.*

- *The capital requirement binds (so $E = \gamma L$) if and only if $R^L < R^E$ or, equivalently, if and only if $\tilde{R}^D(\lambda) < R^E$.*

Proof: See Appendix A.2.

Equation (9) has the interpretation of a zero profit condition. For a bank with a binding capital requirement, one unit of lending is financed by γ in equity and $(1-\gamma)$ in deposits. Thus, competition will equalize the rate of return to lending to the similarly weighted average of the required rates of return of equity and deposits, whence (9). However, deposit finance of loans effectively costs $\tilde{R}^D(\lambda)$ because of the liquidity requirement, which requires that some of the deposits raised must be invested in government bonds.

The all-in cost of financing loans with deposits, $\tilde{R}^D(\lambda)$, exceeds R^D when the return on bonds is less than the interest paid to depositors and it is under that condition, $R^B < R^D$, that the liquidity requirement binds; otherwise, if bonds and deposits yield the same, $\tilde{R}^D(\lambda) = R^D$. ($R^B > R^D$ is ruled out as it is incompatible with a finite solution: it would yield infinite profits as the bank can always raise deposits and invest the proceeds in government bonds, which do not require capital; this stark implication is relaxed in the case of costly financial intermediation.)

The capital requirement binds if equity finance is more expensive than deposit finance, taking into account the impact of the liquidity requirement on the all-in cost of deposit finance. In that situation, the rate on loans will be strictly in between R^E and $\tilde{R}^D(\lambda)$. In contrast, the capital requirement is slack when $R^E = \tilde{R}^D(\lambda)$ and then $R^L = R^E = \tilde{R}^D(\lambda)$, so (9) holds trivially. Regardless of whether the two regulatory constraints are slack or binding – all four cases are possible – economic profits are zero due to the constant returns to scale and perfect competition. Shareholders simply get the competitive return, R^E.

To characterize the more general case with nonnegative intermediation costs, it is convenient to define ρ as the value of the ratio D/L when both regulatory constraints are binding

(in which case $B = \lambda D$ and $E = \gamma L$, so $L + B = E + D$ implies $D/L = (1-\gamma)/(1-\lambda)$):

$$\rho \equiv \frac{1-\gamma}{1-\lambda}$$

Using that, the following proposition summarizes the results for the general case:

Proposition 2 (Solution to the bank's problem when $\lambda \geq w$ and $\phi_\varepsilon \bar{\sigma} \leq r$, so $\sigma = 0$)
A finite solution requires

$$R^B \leq R^D + g_D(D, L) \leq R^L - g_L(D, L) \leq R^E \tag{10}$$

The liquidity requirement binds if and only if the first inequality is strict. The capital requirement binds if and only if the last inequality is strict or, equivalently, if and only if $\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D, L) < R^E$. The solution satisfies the zero-profit condition:

$$R^L - g_L(D, L) = \gamma R^E + (1 - \gamma)\{\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D, L)\} \tag{11}$$

resulting in $\pi^B = 0$. Four cases are possible:

1. *If $R^B = R^E$, then both regulatory requirements are slack, and all relations in (10) hold with equality.*

2. *If $R^B < R^E$, $R^B < R^D + g_D(\rho, 1)$ and $R^L - g_L(\rho, 1) \geq R^E$, then the liquidity requirement binds, so $B = \lambda D$, the capital requirement is slack, and*

$$R^B < R^D + g_D(D, L) < R^L - g_L(D, L) = R^E = \tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(D, L)$$

3. *If $R^B < R^E$, $R^B \geq R^D + g_D(\rho, 1)$ and $R^L - g_L(\rho, 1) < R^E$, then the liquidity requirement is slack, the capital requirement binds, so $E = \gamma L$, and*

$$R^B = R^D + g_D(D, L) < R^L - g_L(D, L) = \gamma R^E + (1 - \gamma)R^B < R^E$$

4. *If $R^B < R^E$, $R^B < R^D + g_D(\rho, 1)$ and $R^L - g_L(\rho, 1) < R^E$, then both regulatory requirements bind, so $B = \lambda D$ and $E = \gamma L$, all inequalities in (10) are strict, and*

$$R^L = \gamma R^E + (1 - \gamma)\tilde{R}^D(\lambda) + g(\rho, 1) \tag{12}$$

Proof: See Appendix A.2.

This closely mirrors proposition 1, except that the interest rates are now adjusted for the marginal resource cost of intermediation. Specifically, the cost of deposit finance

now includes not only the interest rate paid to depositors, R^D, but also the cost of servicing an additional unit of deposits, $g_D(D, L)$. Similarly, the bank deducts the marginal cost of screening and servicing loans, $g_L(D, L)$, from the lending rate that it receives from borrowers. With these adjustments, the conditions for a binding liquidity or for a binding capital requirement remain the same, and competition still equalizes the lending rate to the weighted average of the required return on equity and the all-in cost of financing loans with deposits, with the weights determined by the capital requirement. Because g exhibits constant returns to scale, the marginal zero-profit condition in (11) continues to imply zero total economic profits ($\pi^B = 0$). Also reflecting constant returns to scale, only ratios (not levels) of the balance sheet components are determined by the above optimality conditions.[19] The four cases enumerate when each of the regulatory requirements is binding or slack as a function of variables that the bank takes as given only (that is, excluding D and L).

Zero profits ($\pi^B = rL/R^E - E = 0$) imply that $r = (E/L)R^E \geq \gamma R^E$, so the critical value of $\bar{\sigma}$ for no excessive risk taking ($\sigma = 0$) in (5), r/ϕ_ε, is at least $\gamma R^E/\phi_\varepsilon$, and is equal to that value if the capital requirement binds.[20] Hence,

$$\phi_\varepsilon \bar{\sigma} \leq \gamma R^E \tag{13}$$

is a sufficient condition for no excessive risk taking ($\sigma = 0$), and this condition is also necessary if the capital requirement binds. Note that this condition depends only on variables that the bank takes as given. Working in conjunction with supervision, a sufficiently high capital requirement can deter excessive risk taking by ensuring that the bank internalizes enough of the losses that may arise as a result of such risk taking. The liquidity requirement has no impact on the bank's incentives to engage in excessive risk taking through lending, a point to which I return below.

Liquidity risk choice and solution when $\lambda < w$ As mentioned, analysis of the bank's general problem in (4) differs when $\lambda < w$, as the bank must decide whether or not to self-insure against liquidity problems. That said, many of the details of the analysis are similar to the case $\lambda \geq w$ and so are relegated to Appendix A.3. I focus here on describing and explaining the results.

First, the sufficient condition for no excessive credit risk remains the same as for $\lambda \geq w$ (see (13)):

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[19] The partial derivatives of g are homogenous of degree zero, so $g_D(D, L) = g_D(D/L, 1)$, etc. As discussed below, equilibrium conditions pin down aggregate levels.

[20] $\pi^B = rL/R^E - E$ follows from (7) and the definition of r.

Proposition 3 (Credit risk choice) *For any λ and w,*

$$\phi_\varepsilon \bar{\sigma} \leq \gamma R^E \tag{14}$$

is a sufficient condition for no excessive risk taking ($\sigma = 0$). This condition is also necessary if the capital requirement binds and $B \geq wD$. If $\sigma \neq 0$, then $\sigma = \bar{\sigma}$.

Proof: See Appendix A.3.

An interesting implication is that the liquidity requirement has no impact on the bank's incentives to make excessively risky loans. There are two reasons for this invariance result. First, this type of excessive risk taking occurs through lending, creating credit risk, not liquidity risk. The result may still seem surprising, since the liquidity regulation requires the bank to hold more safe assets (government bonds), which ought to reduce its overall credit risk and the scope for excessive risk taking. However, this is where the second reason comes in: There are no assumptions in the model that artificially limit the level of the bank's total assets. True, a higher liquidity requirement requires the bank to hold more bonds, but the bank does not have to reduce its loan portfolio as a result – it can simply raise more deposits and invest the proceeds in bonds, leaving the scope for excessive risk taking through lending unchanged.

The second result concerns the choice of liquidity risk profile. The basic intuition is as follows. If holding government bonds is costly (in that $R^B < R^D + g_D$) and the probability of liquidity stress is low, then the bank will be tempted to forego the option to self-insure and to set B at its minimum level, λD, which is less than wD and entails the risk of failure due to liquidity stress. This is especially likely when the required liquidity ratio λ is far below the fraction of early withdrawals under stress, w. It is also especially likely if the bank has little equity, so that most of the losses from such an event are borne by the government. Thus, a capital requirement also turns out to improve incentives to self-insure against liquidity risk, in addition to improving incentives with regard to the credit risk profile of the bank.

Formally, the following proposition describes the conditions under which the bank opts to hold liquid assets at a level commensurate with its liquidity risk profile.

Proposition 4 (Liquidity risk choice and solution when $\lambda < w$) *Suppose $\lambda < w$ and $\phi_\varepsilon \bar\sigma \leq \gamma R^E$, so $\sigma = 0$. Let*

$$\zeta \equiv (1 - \gamma)\left(\frac{w}{1 - w} - \frac{\lambda}{1 - \lambda}\right)(R^D - R^B) - \gamma\left(\frac{1 - p}{p}\right)R^E + h$$

If $\zeta \leq 0$ then the bank self-insures against liquidity stress by setting $B \geq wD$ and proposition 7 applies; the bank acts as if $\lambda = w$. If $\zeta > 0$, then the bank sets $B = \lambda D$, is at risk of failure due to liquidity stress, and proposition 8 applies; the bank acts as if the required return on equity is R^E/p.

Proof: See Appendix A.3.

The variable h is defined in equation (38) and collects terms related to differences in intermediation costs between the two business models (that is, $B \geq wD$ versus $B = \lambda D$). For the special case of costless intermediation ($g \equiv 0$), $h = 0$. Propositions 7 and 8 can be found in the appendix and closely mirror proposition 2, with the following exceptions: If $\zeta > 0$, the bank fails if there is an episode of liquidity stress. Moreover, because shareholders get zero in that event, they require that the realized return on equity conditional on such stress *not* occurring is higher by a factor $1/p$. If $\zeta \leq 0$, the bank's behavior is identical to a bank whose liquidity requirement equals w.

As expected, an imprudent liquidity risk profile is especially tempting if the spread $R^D - R^B$ is high, or if the the level of stressed withdrawals w is high relative to the liquidity requirement λ – these factors relate to the cost of self-insurance against liquidity stress. The temptation is smaller if the odds of liquidity stress ($\frac{1-p}{p}$) are high or if the capital requirement γ is high – these factors relate to the expected (private) benefits of insuring against distress from liquidity problems.

A high capital requirement can help incentivize a prudent liquidity risk profile by ensuring that the bank internalizes more of the potential losses from liquidity stress. In addition, even a liquidity requirement that is somewhat below w still has a similar positive incentive effect. Both effects are helpful if (realistically) the exact value of w is not known to the regulator. For example, if the regulator estimates that w is, say, 20 percent, but could be as high as, say, 30 percent for some banks, a 20 percent liquidity requirement could be sufficient if the capital requirement is high enough, avoiding the economic inefficiency of a uniform 30 percent liquidity requirement.

2.3 Firms

Nonfinancial firms cannot create liquidity though deposits. They can, however, produce output of the good using capital and labor as inputs. Capital (K_t) is purchased at the beginning of the period and can be financed by issuing equity to households (E_t^F) and by borrowing from banks (L_t), so $K_t = E_t^F + L_t$.[21]

Firms can employ a riskless or a risky production technology. The riskless technology is standard. Output in period t is $F(K_t, H_t)$, where H_t is hours of labor input and $F()$ is a well-behaved production function exhibiting constant returns to scale. A fraction δ of the capital stock depreciates during the period. Firms last for one period[22] and each period, there is a continuum of firms with mass normalized to one, so each firm takes prices as given.

The firm maximizes shareholder value net of initial equity investment, subject to the constraint that equity cannot be negative:

$$\pi^F = \max_{K,H,E^F \geq 0} \left(F(K,H) + (1-\delta)K - wH - R^L(K - E^F) \right) / R^E - E^F$$

Here loans have been substituted out using the balance sheet identity. The first-order conditions for the choices of labor and capital are standard:

$$(H) \quad F_H(K,H) = w \tag{15}$$

$$(K) \quad F_K(K,H) + 1 - \delta = R^L \tag{16}$$

$$(E^F) \quad R^L/R^E = 1 - \mu, \quad \mu \geq 0, \quad \mu E^F = 0 \tag{17}$$

A finite solution requires $R^E \geq R^L$. If $R^E > R^L$, then $E^F = 0$, so $K = L$. In other words, if bank loans are cheaper than equity finance, the firm chooses to use only bank loans to finance its capital. If $R^E = R^L$, the firm's financial structure is not determined by individual optimality. These optimality conditions, together with the constant returns to scale assumption, imply that economic profits, π^F, equal zero.

Instead of this riskless technology, firms can also choose to use a risky technology, in which case output is $F(K,H) + \sigma_{RF}\varepsilon K$, where ε is the same negative mean, idiosyncratic shock as defined in subsection on loans (and $\sigma_{RF} \geq \bar{\sigma}$). The optimal loan contract with such a firm is the type of risky loan described above, which provides a rationale for capital

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[21] In the model, nonfinancial firms do not want to hold any government bonds as $R^B \leq R^L \leq R^E$ in equilibrium, so explicit consideration of the possibility would not change any of the results.

[22] The absence of adjustment costs and agency problems implies that this is without loss of generality. One can think of ongoing firms as repurchasing their capital stock each period.

regulation (see Van den Heuvel (2008), Appendix B, for details). As mentioned, the analysis will mostly focus on the case that (14) holds, so that banks do not engage in excessive risk taking. No risky firms then exist in equilibrium.

2.4 Government

The government runs fiscal policy, manages the deposit insurance fund, sets capital and liquidity requirements, and conducts bank supervision. The purpose of bank supervision is not only to enforce regulations, but also to monitor excessive risk taking by banks, σ. Supervisors can to some degree detect such behavior and stop any bank that is 'caught' attempting to take on excessive risk in order to protect the deposit insurance fund. It seems reasonable to assume that a small amount of risk taking is harder to detect than a large amount. The largest level of risk-taking that is still just undetectable is $\bar{\sigma}$.

The assumption of imperfect observability of excessive risk taking is important. If regulators could perfectly observe each bank's riskiness, they could simply adjust each bank's deposit insurance premium so as to make the bank pay for the expected loss to the deposit insurance fund, thus eliminating any moral hazard. They could also achieve this by adjusting each bank's capital requirement in response to its true risk. But such perfect observability is simply not realistic, whence the moral hazard problem.[23]

The government's fiscal policy is to maintain a constant level of government debt, $\bar{B}$. T is tax revenue spent on bank supervision. The model allows for a resource cost arising from the resolution of failed banks by the government. $\psi_{Liq} \geq 0$ denotes the deadweight resolution cost per unit of loans in banks that fail early, due to liquidity stress, while $\psi_{Sol} \geq 0$ denotes the resolution cost per unit of loans in banks that fail due to insolvency. Lump-sum taxes are

$$
\begin{aligned}
T_t &= (R_t^B - 1)\bar{B} + T + (1 - p)1_{\{B_t < wD_t\}}(\psi_{Liq} - (r_t - \sigma_t\xi))L_t \qquad (18)\\
&\quad + (1 - (1 - p)1_{\{B_t < wD_t\}})\int_{-\infty}^{-r_t/\sigma_t}(\psi_{Sol} - (r_t + \sigma_t\varepsilon))L_t dF_\varepsilon(\varepsilon)
\end{aligned}
$$

The terms on the right-hand side are respectively the (net) interest on the government

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[23]As in Van den Heuvel (2008), the supervisory bound on σ can be viewed as a risk-based capital requirement or a risk-based deposit insurance premium, but one based on observable risk. Under that interpretation, regulators deter detectable excessive risk taking by imposing a sufficiently high capital requirement, or a sufficiently high deposit insurance premium on that risk when detected.The precise value of this requirement or premium when $\sigma > \bar{\sigma}$ is irrelevant, as it is never implemented in equilibrium. Not inconsistent with this, the model assumes that the bank actually pays a deposit insurance premium equal to zero; this is the actuarially fair deposit insurance premium when (14) holds – the case I focus on.

debt, supervision spending, the cost of resolving banks that fail due to liquidity stress, net of gains/losses from the operation of these banks in resolution by the deposit insurance fund, and the loss to deposit insurance fund due to bank failures associated with loan losses, including deadweight resolution costs.[24] If $\lambda \geq w$ and (14) holds, then taxes are simply: $T_t = (R_t^B - 1)\bar{B} + T$.

2.5 Financial Stability Policy

Optimal regulatory policy involves macroprudential trade-offs. On the benefit side, the capital requirement can deter excessive risk taking by banks, whether from lending to excessively risky borrowers or from holding inadequate buffers of liquid assets. As shown, the liquidity requirement can only address the latter threat to financial stability.

When it happens, banks' excessive risk taking causes a high rate of bank failures, potentially resulting in a financial crisis. Bank failures entail negative externalities: Losses are transferred onto the deposit insurance fund and ultimately to taxpayers. Moreover, this is not a mere transfer from taxpayers to banks, as bank failures and bailouts come with additional deadweight costs and negative spillovers, and their prospect can create distortions ex-ante. In the model, these costs are very simple: the ex-post resolution costs, ψ_{Liq} and ψ_{Sol}, and the ex-ante direct cost of lending to excessively risky firms, ξ. Of course, in reality such costs are vastly more complex.

If these costs are sufficiently high, then it will be socially optimal to deter banks' excessive risk taking, even as the capital and liquidity regulations also entail welfare costs due to reduced liquidity creation by banks, as will be shown very explicitly. Motivated by estimates of the costs of financial crises, I will take the view that avoiding financial crises is in fact socially desirable.

In the model, banks' risk taking choices have dichotomous solutions – that is, either $\sigma = 0$ or $\sigma = \bar{\sigma}$, and liquid assets are either minimal (λD) or high enough to forestall all liquidity stress (at least wD). Therefore, in the model, preventing financial crises means deterring all excessive risk taking. Regulatory policy can accomplish this goal by setting the capital requirement at least as high as the threshold in (14), $\phi_\varepsilon \bar{\sigma}/R^E$, and by ensuring that γ and λ are jointly high enough to incentivize prudent levels of liquid assets ($\zeta \leq 0$; see proposition 4).

However, since $\zeta \leq 0$ results in $B \geq wD$ in any case, *the socially most efficient choice*

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[24]The factor $(1 - 1_{\{B_t < wD_t\}}(1 - p))$ in the last term is included to avoid double-counting resolution costs and credit losses of banks that failed due to liquidity stress *and* had loan losses that would have made them insolvent had they not experienced the liquidity stress.

is then to set $\gamma = \phi_\varepsilon \bar{\sigma}/R^E$ *and* $\lambda = w$. In other words, it is best to rely on the liquidity requirement to ensure a prudent liquidity profile, and let the capital requirement deal with credit risk.[25] This is the case I will mostly focus on in the remainder of this paper.

3 General Equilibrium

Given a government policy λ, γ, and T, an equilibrium is defined as a path of consumption, capital, employment, and financial quantities and returns, for $t = 0, 1, 2, \ldots$, such that households, banks and firms all solve their maximization problems, with and taxes set according to (18), and all markets clear:

$$ e_t = E_t + E_t^F, \quad d_t = D_t, \quad L_t = K_t - E_t^F, \quad B_t + b_t = \bar{B}, \quad H_t = 1 $$

and, for the goods market,

$$ F(K_t, 1) - \xi \sigma_t L_t + (1 - \delta)K_t = c_t + K_{t+1} + g(D_t, L_t) + T + (1 - p)1_{\{B_t < wD_t\}}\psi_{Liq}L_t $$
$$ + (1 - (1 - p)1_{\{B_t < wD_t\}})\psi_{Sol}L_t F_\varepsilon(-r_t/\sigma_t) $$

As explained, I will focus on the case that $\lambda \geq w$ and (14) hold. The government can achieve this by setting λ and γ sufficiently high. In that case, $1_{\{B_t < wD_t\}} = 0$, $\sigma_t = 0$, and $F_\varepsilon(-r_t/\sigma_t) = 0$. By combining this with the market clearing conditions, equations (1), (2), (3), (8), (15), (16) and (17), and proposition 2, the resulting equilibrium allocation can be characterized in terms of a dynamic system in (K_t, c_t) with R_t^E, d_t, b_t, and L_t as auxiliary variables:

$$ K_{t+1} = F(K_t, 1) + (1 - \delta)K_t - c_t - g(d_t, L_t) - T \tag{19} $$

$$ R_t^E = (\beta u_c(c_t, d_t, b_t)/u_c(c_{t-1}, d_{t-1}, b_{t-1}))^{-1} \tag{20} $$

$$ F_K(K_t, 1) + 1 - \delta = R_t^L = R_t^E - \Delta_K(c, d_t, b_t, L_t) \tag{21} $$

with the wedge $\Delta_K = R^E - R^L$ defined as:[26]

$$ \Delta_K(c, d_t, b_t, L_t) \equiv \rho \left(\frac{u_d(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} - g_D(d_t, L_t) - \lambda \frac{u_b(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} \right) - g_L(d_t, L_t) \tag{22} $$

and where d_t, b_t and L_t are determined as follows:

$$ \text{If } \Delta_K(c_t, d_t, b_t, K_t) > 0, \text{ then } L_t = K_t; \text{ else } \Delta_K(c_t, d_t, b_t, L_t) = 0 \tag{23} $$

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[25] To the extent that the quality of supervision can be improved (perhaps at the cost of higher supervision spending), that could help too by reducing the upper bound $\bar{\sigma}$.

[26] The expression for Δ_K follows from (11) and (8): $R^E - R^L = (1 - \gamma)(R^E - \tilde{R}^D(\lambda)) - \rho g_D(D, L) - g_L(D, L) = \rho(R^E - R^D) - \lambda \rho(R^E - R^B) - \rho g_D(D, L) - g_L(D, L)$.

$$d_t = (1 - \gamma)L_t + \bar{B} - b_t \text{ if } \Delta_K(c_t, d_t, b_t, L_t) > 0 \text{ or if } g_L(d_t, L_t) > 0 \tag{24}$$

$$\text{(else } L_t \text{ is indeterminate within } (d_t - \bar{B} + b_t)/(1 - \gamma) \leq L_t \leq K_t)$$

$$\text{If } \Delta_B(c_t, d_t, b_t, L_t) > 0, \text{ then } \bar{B} - b_t = \lambda d_t; \text{ else } \Delta_B(c_t, d_t, b_t, L_t) = 0 \tag{25}$$

with the wedge $\Delta_B = R^D + g_D - R^B$ given by:

$$\Delta_B(c_t, d_t, b_t, L_t) \equiv \frac{u_b(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} - \frac{u_d(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} + g_D(d_t, L_t) \tag{26}$$

The first and second equations restate, respectively, the social resource constraint with $\sigma = 0$ and $\lambda \geq w$ and the household's intertemporal optimality condition (1), which determines the required return on (riskless) equity. In the standard growth model, this rate of return would also equal the marginal product of capital. Here this is not always the case: the marginal product of capital is equated with the banks' lending rate, which can be lower than the cost of equity, as acknowledged by (21).

As in Van den Heuvel (2008), two features of the model are key to understanding how and when this can happen. First, households' liquidity preference implies that the pecuniary return on deposits is lower than the return on equity (see (2)). Second, because of perfect competition in banking, banks will pass on the cheap deposit finance in the form of a lower lending rate.

Now assume for a moment that the capital and liquidity requirements both bind. Then the cheap deposit finance lowers the lending rate by $(1 - \gamma)(R^E - \tilde{R}^D(\lambda))$, as a fraction γ of loans is still financed with bank equity and as the liquidity regulation raises the effective cost of financing loans with deposits by $\tilde{R}^D(\lambda) - R^D = \frac{\lambda}{1-\lambda}(R^D - R^B)$. Using the households' first-order conditions for deposits and bonds and taking into account the noninterest cost of intermediation yields a net reduction in the lending rate, relative to the return on equity, that is equal to Δ_K in equation (22); this is the analogue to the bank's zero profit condition (11).

If this net effect is positive, so that $R^L < R^E$, then firms will rely exclusively on the cheaper bank loans to finance investment and $L = K$ in such a 'pure bank finance' equilibrium. As shown in proposition 2, the capital requirement is in fact binding whenever $R^L < R^E$, and the liquidity requirement binds if and only if the spread $R^D + g_D(d, L) - R^B$ is strictly positive, which in equilibrium is equivalent to $\Delta_B(c, d, b, L) > 0$ (see (26)). In words, the liquidity requirement binds in equilibrium when, at the margin, the convenience premium of Treasuries exceeds, or is not too far below, the marginal value to the liquidity services provided by bank deposits. It follows that, if $\Delta_K(c, \rho K, \bar{B} - \lambda \rho K, K) > 0$ and $\Delta_B(c, \rho K, \bar{B} - \lambda \rho K, K) > 0$, then pure bank finance, along with binding capital and liquidity

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requirements, is indeed an equilibrium. A pure bank finance equilibrium can also occur with a nonbinding liquidity requirement if instead $\Delta_B(c, \rho K, \bar{B} - \lambda \rho K, K) \leq 0$. In that case, the level of bond holdings by banks is determined by the equilibrium condition $\Delta_B(c, (1 - \gamma)K + \bar{B} - b, b, K) = 0$ (see (25)).

Moreover, the equilibrium can also be characterized by 'mixed finance', namely when $\Delta_K(c, d, b, K) < 0$. In that case, firms finance investment with a combination of bank and non-bank funding (so $L < K$). In the model, non-bank funding takes the form of firm equity, but this can be interpreted more broadly as representing any funds raised on capital markets or through non-bank financial intermediaries, including shadow banks. Intuitively, the 'mixed finance' equilibrium prevails when the resource cost of bank intermediation, g, is high relative to the liquidity services of deposits, or because of the combination of a high liquidity requirement, λ, and a high liquidity premium (low yield) on government bonds; see (22) and (23). Thus, if the supply of high quality liquid assets is low relative to demand, the introduction of a very stringent liquidity requirement could cause disintermediation or pressure for activity to shift to shadow banking.

In a mixed finance equilibrium, firms use both equity and bank loans, in such proportion that, in equilibrium, their costs are exactly equal: $R^E = R^L$, and the relative size of the banking sector is then determined endogenously by that condition or, equivalently in equilibrium, by $\Delta_K(c_t, d_t, b_t, L_t) = 0$. Under this condition, the capital and liquidity requirements can each be slack or binding, according to conditions (24) and (25), respectively. Thus, all four cases listed in proposition 2 are possible as part of a 'mixed finance' equilibrium.

In any mixed finance equilibrium, the steady state level of the capital stock satisfies the standard growth model's modified golden rule and is thus independent of any banking variables or liquidity preference. No such a decoupling exists in the pure bank finance equilibrium. In that situation, because banks pass on the low cost of deposits to firms, the steady state capital stock is higher than the modified golden rule's.

Moreover, as a consequence, the steady state levels of the capital stock and income per capita are not invariant to changes in the liquidity requirement or in the capital requirement.[27] With respect to the capital requirement, this non-invariance result is similar to the one obtained in Van den Heuvel (2008), which has been explored more fully in Begenau (2015) and Van den Heuvel (2006). With respect to the liquidity requirement, the non-invariance result is, as far as I know, novel within the context of this type of model.

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[27]See (21) and (22) and recall that $\rho = (1 - \gamma)/(1 - \lambda)$. The result contrasts starkly with the well-known superneutrality result of the Sidrauski (1967) model.

Given that firms in the real world do not rely exclusively on bank loans, it may seem that the mixed finance equilibrium is more realistic, and that the dependence of economic activity in the long run on regulation is a mere theoretical possibility. However, that would be taking the model too literally in my view, as, in reality, there are likely to be some bank-dependent firms and even firms that can access capital markets often have backup lines of credit from banks to facilitate that access.

4   The Welfare Cost of Regulation

To quantify the welfare cost of the liquidity and capital requirements, a social planner's problem will be presented, which is constrained to respect the regulations and devote the same resources to bank supervision. This planner's problem is designed to replicate the decentralized equilibrium, rather than to solve for the first-best. After showing that the planner's allocation is indeed identical to the decentralized equilibrium, this equivalence will then be exploited to derive analytically simple formulas for the welfare costs of the two requirements.[28]

Define the following constrained social planner's problem:

$$V_0(\theta) = \max_{\{c_t, d_t, b_t, L_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, d_t, b_t) \tag{27}$$

$$\text{s.t. } (19), \quad \bar{B} - b_t \geq \lambda d_t, \quad (1 - \gamma) L_t + \bar{B} - b_t \geq d_t, \quad K_t \geq L_t$$

where $\theta = (\lambda, \theta, \bar{\sigma}, T, K_0)$. The constraints correspond to the social resource constraint (for $\sigma = 0$ and $\lambda \geq w$), the liquidity requirement, the capital requirements, and the nonnegativity constraint on firm equity, in that order. Appendix B shows that the allocation associated with this planner's problem is identical to the decentralized equilibrium when regulation satisfies $\lambda \geq w$, so that there are no failures due to liquidity stress, and condition (14) holds for all $t \geq 0$, so that there is no excessive credit risk ($\sigma = 0$). Hence, under these conditions, the constrained social planner's problem replicates the decentralized equilibrium and welfare in that equilibrium is equal to $V_0(\theta)$.

With that, it is straightforward to derive expressions for the marginal welfare cost of the two regulations by differentiating $V_0(\theta)$ with respect to λ and γ, using the envelope theorem. Combining the resulting expressions with the planner's first-order conditions, exploiting the equivalence of the planner's allocation with the decentralized equilibrium's, and using the households' optimality conditions for the choice of deposits and bonds, (2)

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[28]This methodology follows Van den Heuvel (2008).

and (3), yields the main results (see the appendix for details). These are presented in the next two propositions, starting with the liquidity requirement:

Proposition 5 (Gross welfare cost of the liquidity requirement) *Assume that the economy is in steady state in the current period, that $\lambda \geq w$ and that (14) holds. Consider permanently increasing λ by $\Delta\lambda$. A first-order approximation to the resulting welfare loss, expressed as the welfare-equivalent permanent relative loss in consumption, is $\nu_{LIQ}\Delta\lambda$, where*

$$\nu_{LIQ} = \frac{d}{c}\left(R^D + g_D(d,L) - R^B\right)(1-\lambda)^{-1} \tag{28}$$

Proof: See Appendix B.

The above formula is empirically implementable. Remarkably, it does not rely on any assumptions about the functional form of preferences, beyond the standard assumptions of monotonicity, differentiability and concavity. Instead, the formula relies on asset yields to reveal the strength of the household's preference for liquidity. In addition, the measurements presented below will also avoid making any functional form assumptions on the cost function g.

The result shows that there is a positive (gross) welfare cost associated with bank liquidity regulation only to the extent that the interest rate on deposits, plus the marginal cost of servicing deposits, exceeds the interest rate on government bonds. The logic is simple: from the perspective of the other agents, the liquidity requirement effectively forces banks to transform some government bonds into deposits, both instruments prized for their liquidity. Thus, imposing a liquidity requirement entails a social cost only to the extent that the liquidity services of deposits are, at the margin and net of the noninterest cost of creating these services, valued less than those of Treasuries; only then is there a costly *net* reduction in liquidity available to investors. The deposit-Treasury spread, adjusted for the noninterest cost of deposits, reveals whether this is true or not.

The formula takes into account gains and losses associated with the move to a new steady state and is valid whether the equilibrium is characterized by pure bank finance or by mixed bank and nonbank finance and even if the liquidity requirement does not bind (in which case $R^D + g_D(d,L) = R^B$, so $\nu_{LIQ} = 0$ as expected). The regulation is socially costly whenever the requirement binds and has zero cost otherwise.

The next key proposition presents an expression for the marginal gross welfare cost of the capital requirement:

26

Proposition 6 (Gross welfare cost of the capital requirement) *Assume that the economy is in steady state in the current period, that $\lambda \geq w$ and that (14) holds. Consider permanently increasing γ by $\Delta\gamma$. A first-order approximation to the resulting welfare loss, expressed as the welfare-equivalent permanent relative loss in consumption, is $\nu_{CAP}\Delta\gamma$, where*

$$\nu_{CAP} = \frac{L}{c}\left(R^E - \tilde{R}^D(\lambda) - (1-\lambda)^{-1}g_D(d,L)\right) \tag{29}$$

Proof: See Appendix B.

Recall that $\tilde{R}^D(\lambda) \equiv R^D + \frac{\lambda}{1-\lambda}(R^D - R^B)$. Again, the above formula is empirically implementable, does not rely on any assumptions about the functional form of preferences and takes into account gains and losses associated with the move to a new steady state. It is valid whether the equilibrium is characterized by pure bank finance or by mixed bank and equity finance and even if the capital requirement does not bind (in which case $R^E = \tilde{R}^D(\lambda) + (1-\lambda)^{-1}g_D$, so $\nu_{CAP} = 0$ as expected).

An increase in the capital requirement beyond the threshold necessary for financial stability lowers welfare by constraining the ability of banks to issue deposit-type liabilities, which are valued by households for their liquidity. The spread between the risk-adjusted[29] return on bank equity and the pecuniary return on deposits, $R^E - R^D$, reveals the strength of households' preferences for the liquidity services of deposits. However, the production of these services also entails noninterest costs, $g_D(d,L)$, and requires banks to hold more government bonds –which are also prized for their liquidity– in order to satisfy the liquidity requirement. As a consequence, to measure the welfare cost, the simple equity-deposit spread must be adjusted to take these factors into account. Thus, the formula deducts the marginal noninterest cost of deposits[30] and factors in the impact of the liquidity requirement, λ, by using the all-in cost of financing loans with deposits, $\tilde{R}^D(\lambda)$, instead of R^D. Only if a positive spread remains after these adjustment, is a scarcity of deposits due the capital requirement revealed and only then is there a welfare effect at the margin.

This result is generalizes the one obtained in Van den Heuvel (2008). Indeed, proposition 1 in the latter paper, which does not feature liquidity regulation, is nested by setting

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[29]Recall that bank equity is risk-free in the model, so there are no Modigliani-Miller offsets. However, when the model is confronted with the data in the next section, this will be a key area of concern.

[30]The factor $1/(1-\lambda)$ multiplying $g_D(d,L)$ reflects the fact that the bank must raise $1/(1-\lambda)$ in deposits to finance one unit of lending, while satisfying the liquidity requirement.

$\lambda = 0$ in (29):[31]

$$\nu_{CAP}|_{\lambda=0} = (L/c)\left(R^E - R^D - g_D(d, L)\right) \qquad (30)$$

Relatedly, the impact of the liquidity requirement on the welfare cost of the capital requirement can be seen more explicitly by rewriting (29) using (28) and (8):

$$\nu_{CAP} = (L/c)\left(R^E - R^D - g_D(d, L)\right) - (L/d)\lambda\nu_{LIQ}$$

Thus, *for given observables* (R^E, R^D, etc.) imposing a liquidity requirement lowers the welfare cost of the capital requirement if the liquidity requirement binds. Of course, these observables are generally not completely invariant to changes in the liquidity requirement.

5 Measurement of the Welfare Costs

The goal of this section is to measure the gross welfare costs of bank liquidity requirements and bank capital requirements by combining the formulas derived in the previous section with data. To that end, I use annual aggregate balance sheet and income statement data for all FDIC-insured commercial banks in the United States. These data are obtained from the FDIC's Historical Statistics on Banking (HSOB) and are based on regulatory filings ('call reports'). I employ data from the period 1986 to 2013.[32]

A key challenge to the empirical application of the formulas in (28) and (29) is the measurement of the marginal net noninterest cost of servicing deposits, g_D. This includes the cost of ATMs, some of the cost of maintaining a network of branches, etc. Fees on deposits should be netted out. The call reports contain data on noninterest expense and revenue, and the difference, net noninterest cost, is nontrivial, averaging 1.31 percent of total assets on an annual basis over the 1986-2013 period. However, there is little information in the data permitting a breakdown by activity (e.g. servicing deposits, screening loan applications, collecting payments, etc.).
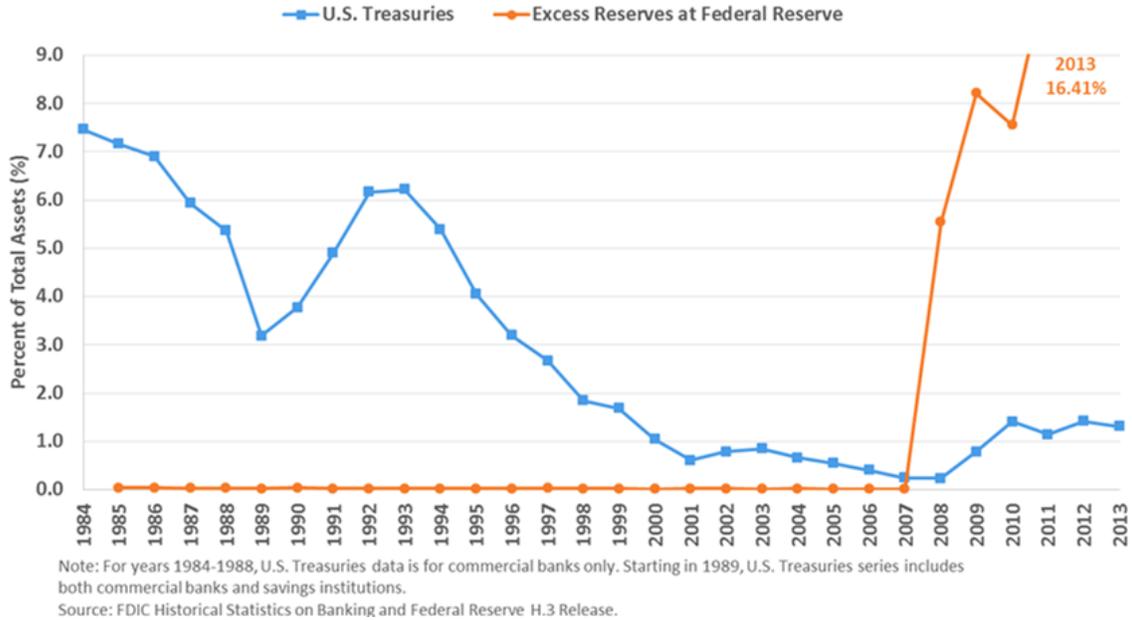
Fortunately, however, the model suggests a way to infer the marginal net noninterest cost of deposits when banks voluntarily hold Treasuries on their balance sheet. Specifically, proposition 2 shows that whenever the liquidity requirement is not binding, then

$$R^D + g_D(d, L) = R^B \qquad (31)$$

---

[31] The statement of proposition 1 in Van den Heuvel (2008) also uses the fact that, in that model, $L = d/(1-\gamma)$ if $R^E - R^D - g_D(d, L) \neq 0$.

[32] Regulation Q, which placed restrictions on banks' deposit rates, was fully phased out on January 1, 1986.

Figure 1. U.S. Treasuries and Excess Reserves held by U.S. Depository Institutions
(Share of Total Assets, 1984-2013)

Note: For years 1984-1988, U.S. Treasuries data is for commercial banks only. Starting in 1989, U.S. Treasuries series includes both commercial banks and savings institutions.
Source: FDIC Historical Statistics on Banking and Federal Reserve H.3 Release.

The interpretation is banks will only hold more Treasuries than required if investing in Treasuries and financing them with deposit-type liabilities is not a money-losing activity, taking into account noninterest costs. Thus, by finding a historical period when banks held Treasuries well in excess of any regulatory requirements, we can infer g_D from that period's Treasury-deposit spread.

Figure 1 shows U.S. Treasuries and excess reserves held by U.S. depository institutions, expressed as a share of total assets, from 1986 to 2013.[33] As can be seen from the chart, between 1986 and 2000, banks invested a significant part of their balance sheet in Treasuries (more than 1 percent of total assets). This asset allocation was voluntary, as there was no Basel-style liquidity requirement applicable during the entire period covered by the chart (so $\lambda = 0$ in the sense of the model).[34] Reserve requirements were in place, but these could

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[33]In a sense, excess reserves can be thought of as holding Treasuries through the Federal Reserve's balance sheet. However, excess reserves were not renumerated by the Fed until 2008, when excess reserves started to grow rapidly.

[34]One might view (parts of) the sample period as one where liquidity regulation was unnecessary either because high quality liquid assets were abundant so banks voluntarily held sufficient liquid assets, or because there was an effective lender of last resort and interbank market. Alternatively, one might view (the more recent part of) the sample period as one where liquidity regulation was in fact necessary and the financial crisis that started in 2007 was in part the result of its absence.

only be satisfied by holding reserves at the Fed, not by holding Treasuries. Thus, I use data from the 1986-2000 period to infer g_D using equation (31).

For R^B, I use the 3-month Treasury bill rate on the secondary market. The average net interest rate on deposits, $R^D - 1$, is calculated as the HSOB's Interest on Total Deposits divided by Total Deposits.[35] For the period 1986-2000, the resulting average Treasury-deposit spread, $R^B - R^D$, equals 1.22 percent. Accordingly, I set the marginal net noninterest cost associated with deposits at $g_D = 0.0122$ per annum.

It is useful to compare this to an independent, upper bound estimate. The upper bound is obtained by attributing all net noninterest cost to servicing deposits and none to lending. Maintaining the assumption of constant returns to scale of g, this would imply that $g_D(D, L) = g(D, L)/D$.[36] The latter ratio is equal to 0.0216 per annum, on average for the same time period. Consistent with the model, this upper bound exceeds the spread-based estimate and suggests that a bit more than half of total net noninterest cost can be attributed to deposits, an implication that strikes me as plausible.

To map the data into the remaining variables, I largely follow Van den Heuvel (2008). For deposits, D, the HSOB's Total Deposits is used. For consumption, c, personal consumption expenditures from the NIPA is used. For loans, I use Total Assets net of U.S. Treasuries and excess reserves. To quantify the welfare costs, I calculate long run averages of the ratios and the spreads in the formulas in propositions 5 and 6, starting with the welfare cost of liquidity requirements.

5.1   Liquidity regulation

It is important to note at the outset that banks presently already have elevated stocks of high quality liquid assets, so that a moderate liquidity requirement would be nonbinding for most banks. In large part, this reflects the high level of excess reserves, documented in figure 1, which increased in 2008 and following the Federal Reserve's large scale asset purchases in 2009 and 2011, and have remained high in an environment of very low interest rates and payment of interest on excess reserves.[37] Thus, *in the current environment, the introduction of a moderate liquidity requirement would likely entail little or no immediate economic cost.*

---

[35] All data are nominal. While the model is real, using nominal data consistently is correct, because the formulas for the welfare costs contain only ratios of quantities and spreads of returns.

[36] Linear homogeneity of g implies $g(D, L) = g_D(D, L)D + g_L(D, L)L \geq g_D(D, L)D$.

[37] See Ennis and Wolman (2015) for an emprirical analysis of banks' excess reserves. Among other findings, they document that the reserves are widely distributed across banks.

However, the new liquidity rules, the LCR and the NSFR, are intended to be structural, long-run measures, and it is difficult to predict for how long banks would voluntarily maintain their large liquidity buffers in the absence of such regulation. As a result, the question of long-run economic costs and benefits of liquidity regulation remains a live one, even if modest requirements would be nonbinding for most banks in the current environment.

To address the question of long-run economic costs, I use data from a period when liquidity requirements would have been more likely to bind. As can be seen in figure 1, in each year between 2001 and 2007, banks' holding of Treasuries plus excess reserves were less than 1 percent of total assets. Thus, this period is a good candidate to gauge the potential welfare cost of liquidity regulation.

Over 2001-2007, the average nominal yields on Treasuries and deposits are, respectively, 2.80% and 2.04%, so the average spread is 76 basis points, less than the marginal noninterest cost of servicing deposits, which we have already estimated at 122 basis points. The mean deposit to consumption ratio is 0.67, and, as already noted, there was no liquidity requirement in place ($\lambda = 0$). Applying (28), the first-order approximation to the gross welfare of introducing a liquidity requirement set at $\lambda_{new} > 0$ is:

$$
\begin{aligned}
\nu_{LIQ}\lambda_{new} &= \frac{d}{c}\left(R^D + g_D(d,L) - R^B\right)(1-\lambda)^{-1}\lambda_{new} \\
&= 0.67 \times (0.0122 - 0.0076) \times 1 \times \lambda_{new} = 0.0031 \times \lambda_{new}
\end{aligned}
$$

To interpret this number, consider the gross welfare cost of a 10 percent liquidity requirement. This is equivalent to a permanent loss in consumption of

$$
\nu_{LIQ} \times 0.1 = 0.0031 \times 0.1 \times 100\% = 0.031\%
$$

or about \$3.5 billon per year (using 2013 consumption). While perhaps not trivial, compared to estimates of the welfare cost of inflation (see e.g. Lucas (2000)), or compared to many existing estimates of the welfare cost of capital requirements (see e.g. BCBS (2010)), this is a relatively small number. Even modest financial stability benefits would easily justify this cost.[38]

As a caveat, this estimate is a first-order approximation and may be less accurate for large changes in liquidity regulation. In particular, it is possible that larger increases in the liquidity requirement would entail more than proportionally larger welfare costs, as the stock of Treasuries that remains available to the non-bank public progressively shrinks.

---

[38]It is interesting to note that the cost is non-negative, as predicted by the model. There is nothing in the empirical methodolgy that guaranteed a non-negative number, so this might be viewed as a small empirical validation of the the model.

5.2 Capital regulation

To measure the welfare cost of capital requirements, we need an accurate estimate of the required return on (bank) equity, and the degree to which that required return adjusts in response to changes in bank leverage that would be brought about by altering capital requirements. In particular, for given asset risk, a decline in leverage should make bank shares less risky, and in theory this should lower the required return that shareholders demand. Indeed, under the idealized conditions underlying Modigliani and Miller's propositions, the strength of this effect is just such that the weighted average cost of funds for the firm does not depend on its leverage at all.

In reality, there are several reasons why the Modigliani-Miller theorem does not hold – agency problems, taxes, bankruptcy costs, etc. – and it is especially unlikely to hold for banks in light of the special nature of their debt. Indeed, in the model presented, the liquidity of bank debt is simultaneously the reason that banks exist and the reason the Modigliani-Miller theorem fails to hold for them. On this point, empirical analysis by Baker and Wurgler (2014) finds that, while better-capitalized banks have lower risk as expected, lower-risk banks tend to have higher stock returns on a risk-adjusted or even raw basis, so that an increase in capital ratios would result in a sharply higher weighted average cost of capital, an outcome that would be qualitatively consistent with the model. Nonetheless, even if the Modigliani-Miller theorem does not hold exactly in the model and in reality, it is still likely that the expected return on equity adjusts to changes in bank leverage, and the empirical approach should take this into account.

Thus, whereas the model abstracts from aggregate risk, a risk-adjusted measure of the required return on equity is called for from the data. Following Van den Heuvel (2008), I use the average return on subordinated bank debt as a proxy for the risk-adjusted return on equity. The reason for this choice is that (a) subordinated debt counts towards regulatory equity capital, albeit within certain limits, and (b) defaults on this type of debt have historically been rare, so the debt is not very risky, certainly much less risky than common equity. This proxy avoids the difficulties inherent in measuring the (ex ante) risk premium on common equity,[39] and how that premium adjusts to changes in leverage. Concretely, $(R^E - 1)$ is measured by Interest on Subordinated Notes divided by Subordinated Notes.[40]

---

[39] For example, the historical average excess return on bank equity would imply a high premium, but does this equal the *ex ante* expected premium? In addition, depending on what interest rate is used to measure the excess return on equity, this approach would run the risk of contaminating the measured risk premium with a liquidity premium, which one would definitely want to avoid in the present context.

[40] Part of the HSOB's Subordinated Notes does not qualify as regulatory capital. However, cross-checking

The limits on the use of subordinated debt for regulatory purposes imply that this is a conservative measure for the risk-adjusted required return on bank equity. First, subordinated debt can count only towards tier 2 capital, so it only helps to satisfy the risk-based total capital ratio requirement, not the risk-based tier 1, common equity tier 1, or leverage ratio requirements. Second, until the recent adoption of Basel III, the amount of subordinated debt in tier 2 was limited to 50 percent of the bank's tier 1 capital. So if the tier 1 capital ratio was close to binding, subordinated debt could count for at most approximately 25 percent of total capital. Due to these limits, it is possible that for many banks the required return on subordinated debt is lower than the risk-adjusted return on regular equity.

To quantify the welfare cost of capital requirements using subordinated debt, I use 1993-2010 as a preferred sample period, because the Basel Accord and the FDICIA enacting it were not fully implemented until January 1, 1993, and prior to Basel the use of subordinated debt for regulatory purposes was rather limited. 2010 is chosen as end date, because the Basel III package was published in December 2010. However, extending the sample through 2013, or letting start earlier, has little impact on the results.

For 1993-2010, the average nominal net returns on subordinated debt and deposits are, respectively, 5.45% and 2.43%, so the average spread is 302 basis points. The mean loans to consumption ratio is 0.96 (using total assets minus Treasuries and excess reserves for loans). As explained, the net noninterest cost of servicing deposits is set at 122 basis points and $\lambda = 0$ for this period. Combining these measurements with the analytical result in (30) yields a marginal gross welfare cost of the capital requirement equal to

$$
\begin{aligned}
\nu_{CAP} &= (L/c)\left(R^E - R^D - g_D(d, L)\right) \\
&= 0.96 \times (0.0302 - 0.0122) = 0.0173
\end{aligned}
$$

Thus, the gross welfare cost of an increase in the capital requirement by 10 percentage points is equivalent to a permanent loss in consumption of

$$
\nu_{CAP} \times 0.1 \times 100\% = 0.17\%
$$

Of course, this should be compared to the financial stability benefits of such an increase. In the model, those benefits are (in part by assumption) very large initially, but then they abruptly drop to zero once the threshold required to safeguard financial stability is

with the call reports (item RCFD5610) indicates that the difference is minimal after 1992. Also, some subordinated bank debt is callable. Flannery and Sorescu (1996) find that the average call option value for callable bank sub-debt is 0.19%, so the point is minor for the present purpose.

exceeded (see 14). Of course, in reality the relation between capital requirements and financial stability is likely to be smoother.

Compared to the welfare cost of a similarly sized liquidity requirement, this is about five times as large. Thus, although capital requirements have broader financial stability benefits in the model than liquidity requirements (which only address liquidity risk taking), their welfare costs are also substantially larger according to these estimates.

This reflects an insight from the model: capital requirements reduce liquidity creation by banks much more than liquidity requirements do. Even with the general equilibrium feedbacks that change the size of the banking sector, capital requirements effectively reduce the supply of bank deposits, replacing them to some degree with bank equity, an instrument that does not provide liquidity services. In contrast, liquidity requirements effectively transform some government bonds held by the public into bank deposits. However, these are both liquid instruments that command a convenience yield, so the *net* reduction in liquidity is much smaller. In the data, this is evidenced by a smaller spread between Treasuries and bank deposits than between equity and deposits, and that is why the welfare cost estimates differ as much as they do.

Table 1. Gross Welfare Costs of Liquidity and Capital Requirements

(welfare-equivalent permanent consumption loss in percent)

Welfare cost of:	Sample	
	2001-2007	1993-2010
10% liquidity requirement	0.031	0.024
10% capital requirement	0.181	0.173

Finally, for comparison, table 1 reports estimates of the gross welfare costs of both regulations based on both sample periods. As can be seen, the cost of capital requirements does not vary much at all depending on the time period considered. The cost of a 10% liquidity requirement is somewhat lower when using data from 1993-2010. This is not surprising since this longer sample includes periods when a modest liquidity requirement would likely have been nonbinding for most banks, as noted previously.

6   Conclusion

I have presented a framework for measuring the welfare effects of bank liquidity and capital requirements. While such requirements have important financial stability benefits, they also

entail social costs because they reduce banks' ability to create net liquidity in equilibrium. Using U.S. data, the welfare cost of a 10 percent liquidity requirement is found to be equivalent to a permanent loss in consumption of about 0.03%, a modest impact. According to conservative estimates, the cost of a 10 percentage point increase in capital requirements is about 0.17%, significantly larger. However, the financial stability benefits of capital requirements were found to be broader than those of liquidity requirements. Liquidity requirements are not a substitute for capital requirements.

Appendix A. Analysis of the bank's problem

A.1. Excessive risk taking, part 1

Since loans are nonnegative, expected dividends equal $E\left[\{(r+\sigma\varepsilon)\}^{+}\right]L$. The expectation expectation equals

$$
\begin{aligned}
\mathbb{E}\left[\{(r+\sigma\varepsilon)\}^{+}\right] &= (r+\bar{\sigma}\mathbb{E}\varepsilon)-\mathbb{E}\left[\{r+\bar{\sigma}\varepsilon\}^{-}\right] \\
&= (r-\sigma\xi)+\sigma j(-r/\sigma)
\end{aligned}
$$

where j is the following function, derived from the distribution function of ε :

$$
j(x) \equiv \int_{\underline{\varepsilon}}^{x}(x-\varepsilon)dF_{\varepsilon}(\varepsilon)
$$

Note that $j(x)$ is continuous and increasing in x, equals zero when $x=\underline{\varepsilon}$ and strictly exceeds $\xi \geq 0$ when $x=0$ (by the definition of ξ and the assumption $\bar{\varepsilon}>0$). As shown in Van den Heuvel (2008), appendix D.1, $\sigma j(-r/\sigma)$ is a convex function of σ. Hence, expected dividends are convex in σ. Therefore, either $\sigma=0$ or $\sigma=\bar{\sigma}$ is optimal. Comparing these two choices,

$$
\sigma = 0 \ \ \text{if and only if} \ \ j(-r/\bar{\sigma}) \leq \xi
$$

The definition of ϕ_{ε} in (6) can be rewritten in this notation, as follows:

$$
j(-\phi_{\varepsilon}) \equiv \xi
$$

From the above-mentioned properties of j, it follows that ϕ_{ε} exists, is unique and satisfies $\underline{\varepsilon} \leq -\phi_{\varepsilon}<0$, so $0<\phi_{\varepsilon}\leq-\underline{\varepsilon}$. Using this notation, $\sigma=0$ if and only if $-r/\bar{\sigma}\leq-\phi_{\varepsilon}$ or equivalently, $\phi_{\varepsilon}\bar{\sigma}\leq r$. Otherwise, as already noted, $\sigma=\bar{\sigma}$. [41] This is result in (5).

---

[41] When $\phi_{\varepsilon}\bar{\sigma}=r$, the bank is indifferent. For convenience, it is assumed that $\sigma=0$ in that case.

A.2. The bank's problem when $\lambda \geq w$ and $\phi_\varepsilon \bar{\sigma} \leq r$, so $\sigma = 0$ (proof of propositions 1 and 2)

After scaling the problem in (7) by R^E and using the balance sheet identity to substitute out B, the lagrangian and first-order conditions (FOCs) are:[42]

$$\mathcal{L} = R^L L + R^B(E + D - L) - R^D D - g(D, L) - R^E E$$
$$+ \Lambda[E + (1 - \lambda)D - L] + \chi[E - \gamma L]$$

$$\begin{aligned}
(L) \quad R^L &= R^B + g_L + \Lambda + \gamma \chi \\
(E) \quad R^E &= R^B + \Lambda + \chi \\
(D) \quad R^D + g_D &= R^B + (1 - \lambda)\Lambda
\end{aligned}$$

The complementary slackness conditions are: $\chi[E - \gamma L] = 0$, $\chi \geq 0$, $\Lambda[E + (1 - \lambda)D - L] = 0$, $\Lambda \geq 0$. Note that

$$R^B + \Lambda = R^D + g_D + \lambda\Lambda = R^L - g_L - \gamma\chi = R^E - \chi \tag{32}$$

Since the Kuhn-Tucker multipliers must be nonnegative, a finite solution requires the ranking of returns shown in (10) in proposition 2, i.e.:

$$R^B \leq R^D + g_D \leq R^L - g_L \leq R^E$$

From FOC (D)

$$\Lambda = \frac{1}{1 - \lambda}(R^D + g_D - R^B)$$

Hence, the liquidity requirement binds if and only if $R^D + g_D > R^B$.

In addition, from (32),

$$\chi = \frac{1}{1 - \gamma}(R^E - (R^L - g_L))$$

Hence, the capital requirement binds if and only if $R^E > R^L - g_L$.

Furthermore, (32) implies

$$R^L - g_L - \gamma\chi = \gamma\{R^E - \chi\} + (1 - \gamma)\{R^D + g_D + \lambda\Lambda\}$$

Rearranging and using the expression for Λ as well as the definition of $\tilde{R}^D(\lambda)$ (see (8)) yields the (marginal) zero-profit condition:

$$R^L - g_L(D, L) = \gamma R^E + (1 - \gamma)\{\tilde{R}^D(\lambda) + \frac{1}{1 - \lambda} g_D(D, L)\} \tag{33}$$

---

[42] For brevity, the arguments of $g(D, L)$ and its partial derivatives are often suppressed where this does not lead to confusion.

which is (11) in proposition 2. Furthermore, this implies that $R^L - g_L - R^E = (1 - \gamma)[\{\tilde{R}^D(\lambda) + \frac{1}{1-\lambda}g_D\} - R^E]$, yielding the equivalent, alternative condition for a (non)binding captial requirement:

$$R^L - g_L(D, L) < (=) \; R^E \quad \Longleftrightarrow \quad \tilde{R}^D(\lambda) + \frac{1}{1-\lambda}g_D(D, L) < (=) \; R^E$$

Zero profits follow from

$$
\begin{aligned}
R^E \pi^B &= R^L L + R^B B - R^D D - g(D, L) - R^E E \\
&= R^L L + R^B (E + D - L) - R^D D - (Dg_D(D, L) + Lg_L(D, L)) - R^E E \\
&= [R^L - g_L(D, L) - R^B]L - [R^D + g_D(D, L) - R^B]D - [R^E - R^B]E \\
&= [\Lambda + \gamma\chi]L - [(1 - \lambda)\Lambda]D - [\Lambda + \chi]E \\
&= -\chi(E - \gamma L) - \Lambda[E + (1 - \lambda)D - L] \\
&= 0
\end{aligned}
$$

where the steps follow from Euler's theorem, the first-order conditions and the complementary slackness conditions, in that order.

This concludes the proof of the first half of proposition 2 (up to 'four cases are possible'). Proposition 1 in the main text follows immediately as a corollary by setting $g = 0$ in the first half of proposition 2. The second half of proposition 2 (after 'four cases are possible') characterizes the solution further by showing when each of the two regulatory constraints is binding or slack as a function of objects that the bank takes as given only. The issue is that the conditions derived so far – that is, $R^D + g_D(D, L) > R^B$ for a biding liquidity requirement and $R^E > R^L - g_L(D, L)$ for a binding capital requirement – still depend on decision variables of the bank, namely D and L, so that the characterization of the solution is not complete. (This issue does not arise when $g = 0$, the case summarized in proposition 1.)

Recall that ρ is defined as the value of D/L when both regulatory constraints are binding: $\rho = \frac{1-\gamma}{1-\lambda}$.

Case 1. Nonbinding constraints ($\chi = 0$ and $\Lambda = 0$) From the FOCs,

$$R^B = R^D + g_D = R^L - g_L = R^E$$

Note that this case requires $R^D \le R^B = R^E \le R^L$. As g is homogenous of degree 1, its partial derivatives are homogenous of degree 0. Consequently, the ratio D/L is determined by $g_D(D/L, 1) = R^B - R^D$ and by $g_L(D/L, 1) = R^L - R^B$. A solution to this case requires

that the configuration of returns is such that these two equations imply the same value for D/L. With both constraints nonbinding, D/L may be higher or lower than ρ.

Case 2. Only liquidity requirement binds ($\chi = 0$ and $\Lambda > 0$) From the FOCs, $R^L - g_L(D, L) = R^E$. From this and (33) it follows that

$$R^L - g_L(D, L) = R^E = \tilde{R}^D(\lambda) + \frac{1}{1 - \lambda} g_D(D, L)$$

Again, we have two equations that each pin down the ratio D/L. This case requires that $R^B < R^D + g_D(D, L) < R^L - g_L(D, L) = R^E$ and in particular $R^B < R^E$. Due to the nonbinding capital requirement, $D/L \leq \rho$. Hence, as g is convex, $g_D(D, L) = g_D(D/L, 1) \leq g_D(\rho, 1)$ and $g_L(D, L) = g_L(D/L, 1) \geq g_L(\rho, 1)$ and

$$R^B < R^D + g_D(\rho, 1)$$
$$R^L \geq R^E + g_L(\rho, 1)$$

Case 3. Only capital requirement binds ($\chi > 0$ and $\Lambda = 0$) From the FOCs,

$$R^B = R^D + g_D$$

and

$$\begin{aligned} R^L - g_L &= \gamma R^E + (1 - \gamma)(R^D + g_D) \\ &= \gamma R^E + (1 - \gamma) R^B \end{aligned}$$

Note that this case requires that $R^B = R^D + g_D < R^L - g_L < R^E$. Due to the nonbinding liquidity requirement, $D/L \geq \rho$. Consequently, $g_D(D, L) \geq g_D(\rho, 1)$ and $g_L(D, L) \leq g_L(\rho, 1)$ and

$$R^B \geq R^D + g_D(\rho, 1)$$
$$R^L < R^E + g_L(\rho, 1)$$

Also,

$$R^D + g_D(\rho, 1) < R^E$$

Case 4. Both requirements bind ($\chi > 0$ and $\Lambda > 0$) In this case, $D/L = \rho$, so (33) implies $R^L - g_L(\rho, 1) = \gamma R^E + (1 - \gamma)\{\tilde{R}^D(\lambda) + \frac{1}{1-\lambda} g_D(\rho, 1)\}$. Using Euler's theorem and $\rho = (1 - \gamma)/(1 - \lambda)$, this can also be written as

$$R^L = \gamma R^E + (1 - \gamma)\tilde{R}^D(\lambda) + g(\rho, 1)$$

which is (12). Note that $g(\rho, 1)$ is the total resource cost of making one unit of loans and financing it with ρ units of deposits. With $\chi > 0$ and $\Lambda > 0$, the inequalities in the ranking of returns (10) are now all strict. Moreover, because $D/L = \rho$, we have

$$R^B < R^D + g_D(\rho, 1) < R^L - g_L(\rho, 1) < R^E$$

This concludes the proof of proposition 2. QED.

(The reader may wonder why the case $R^B < R^E$, $R^B \geq R^D + g_D(\rho, 1)$ and $R^L - g_L(\rho, 1) \geq R^E$ is missing from proposition 2. The reason is that this configuration is incompatible with a finite solution. To see why, consider a bank that makes feasible choices $B = \lambda D$ and $E = \gamma L$. Its profits π^B would be $\{R^L - \gamma R^E - (1-\gamma)\tilde{R}^D(\lambda) - g(\rho, 1)\}L/R^E = \{R^L - \gamma R^E - (1-\gamma)\tilde{R}^D(\lambda) - (\rho g_D(\rho, 1) + g_L(\rho, 1)\}L/R^E = \{R^L - g_L(\rho, 1) - \gamma R^E - (1-\gamma)(R^D + \frac{\lambda}{1-\lambda}(R^D - R^B)) - \frac{1-\gamma}{1-\lambda}g_D(\rho, 1)\}L/R^E =$

$$\{\underbrace{R^L - g_L(\rho, 1) - R^E}_{\geq 0} + (1-\gamma)(\underbrace{R^E - (R^D + g_D(\rho, 1))}_{>0}) + \frac{\lambda}{1-\lambda}\underbrace{(R^B - (R^D + g_D(\rho, 1)))}_{\geq 0})\}L/R^E,$$

where the strict inequality follows from $R^E > R^B \geq R^D + g_D(\rho, 1)$. Thus, profits would be strictly and linearly increasing in L and the bank would want to have infinite scale.)

A.3. The Bank's problem when $\lambda < w$ (proof of propositions 3 and 4)

Recall that if $\lambda \geq w$ then any feasible choice implies $1_{\{B < wD\}} = 0$, resulting in the problem (cf. (4))

$$\pi^B|_{\lambda \geq w} = \max_{\sigma, L, B, D, E} \quad \mathbb{E}\left[\{(R^L + \sigma\varepsilon)L + R^B B - R^D D - g(D, L)\}^+\right]/R^E - E$$
$$\text{s.t.} \quad L + B = E + D, \quad B \geq \lambda D, \quad E \geq \gamma L, \quad \sigma \in [0, \bar{\sigma}]$$

To analyze the case $\lambda < w$, we make use of the mathematical fact that if $S = A \cup B$, then $\max_{x \in S} f(x) = \max\{\max_{x \in A} f(x), \max_{x \in B} f(x)\}$, provided $\max_{x \in A} f(x)$ and $\max_{x \in B} f(x)$ both exist. Thus,

$$\pi^B|_{\lambda < w} = \max\{\pi^B|_{B < wD}, \pi^B|_{B \geq wD}\}$$

where

$$\pi^B|_{B \geq wD} = \max_{\sigma, L, B, D, E} \quad \mathbb{E}\left[\{(R^L + \sigma\varepsilon)L + R^B B - R^D D - g(D, L)\}^+\right]/R^E - E$$
$$\text{s.t.} \quad L + B = E + D, \quad B \geq wD, \quad E \geq \gamma L, \quad \sigma \in [0, \bar{\sigma}]$$

and

$$\pi^B|_{B < wD} = \max_{\sigma, L, B, D, E} \quad \mathbb{E}\left[(1-\eta)\{(R^L + \sigma\varepsilon)L + R^B B - R^D D - g(D, L)\}^+\right]/R^E - E$$
$$\text{s.t.} \quad L + B = E + D, \quad B \in [\lambda D, wD), \quad E \geq \gamma L, \quad \sigma \in [0, \bar{\sigma}]$$

which, using the independence of ε and η can also be written as

$$\pi^B|_{B<wD} = \max_{\sigma,L,B,D,E} \quad p\mathbb{E}\left[\{(R^L + \sigma\varepsilon)L + R^B B - R^D D - g(D,L)\}^+\right]/R^E - E$$
$$\text{s.t.} \quad L + B = E + D, \quad B \in [\lambda D, wD), \quad E \geq \gamma L, \quad \sigma \in [0, \bar{\sigma}]$$

The strict inequality constraint $B < wD$ could lead to nonexistence of a solution to this problem. However, we will show below that this issue does not arise. In addition, it turns out to be mathematically convenient to define a slightly modified problem (which differs only in the liquidity constraint):

$$\pi^B|_{liq.\,risk} = \max_{\sigma,L,B,D,E} \quad p\mathbb{E}\left[\{(R^L + \sigma\varepsilon)L + R^B B - R^D D - g(D,L)\}^+\right]/R^E - E$$
$$\text{s.t.} \quad L + B = E + D, \quad B \geq \lambda D, \quad E \geq \gamma L, \quad \sigma \in [0, \bar{\sigma}]$$

(The subscript *liq. risk* stands for *liquidity risk*.) Note that $\pi^B|_{liq.\,risk} \geq \pi^B|_{B<wD}$ because the set the of feasible choices is larger. However, if $\pi^B|_{liq.\,risk} > \pi^B|_{B<wD}$, it must be because the optimal choice involves $B \geq wD$ and for any such choice $\pi^B|_{liq.\,risk} \leq \pi^B|_{B\geq wD}$ (as $p < 1$). Hence,

$$\pi^B|_{\lambda<w} = \max\{\pi^B|_{liq.\,risk}, \pi^B|_{B\geq wD}\}$$

Note that these latter two problems are isomorphic to $\pi^B|_{\lambda \geq w}$:

$\pi^B|_{B\geq wD}$ is identical to $\pi^B|_{\lambda \geq w}$ if λ is set equal to w in the latter

$\pi^B|_{liq.\,risk}$ is identical to $\pi^B|_{\lambda \geq w}$ if R^E replaced by R^E/p in the latter

Credit risk choice with $\lambda < w$, part 1

Recall that the choice of σ conditional on L, B, D, E for problem $\pi^B|_{\lambda \geq w}$ was determined from the maximization of $\mathbb{E}\left[\{(r + \sigma\varepsilon)L\}^+\right]$, where $r \equiv R^L + R^B(B/L) - R^D(D/L) - g(D/L, 1) > 0$. Also, recall that problem $\pi^B|_{B\geq wD}$ differs from $\pi^B|_{\lambda \geq w}$ only in that λ is replaced with w in the liquidity constraint. Thus, for problem $\pi^B|_{B\geq wD}$, the optimal choice of σ conditional on L, B, D, E also maximizes $\mathbb{E}\left[\{(r + \sigma\varepsilon)L\}^+\right]$ subject to $\sigma \in [0, \bar{\sigma}]$.

Now consider problem $\pi^B|_{liq.\,risk}$. In this notation,

$$\pi^B|_{liq.\,risk} = \max_{\sigma,L,B,D,E} \quad p\mathbb{E}\left[\{(r + \sigma\varepsilon)L\}^+\right]/R^E - E$$
$$\text{s.t.} \quad L + B = E + D, \quad B \geq \lambda D, \quad E \geq \gamma L, \quad \sigma \in [0, \bar{\sigma}]$$

Again, since p is strictly positive the optimal choice of σ conditional on L, B, D, E maximizes $\mathbb{E}\left[\{(r + \sigma\varepsilon)L\}^+\right]$ subject to $\sigma \in [0, \bar{\sigma}]$.

Hence, the condition for $\sigma = 0$ shown in (5) also applies to $\pi^B|_{B\geq wD}$ and $\pi^B|_{liq.\,risk}$. Therefore:

Lemma 1 *Regardless of the values of λ and w, $\sigma = 0$ if and only if $\phi_\varepsilon \bar{\sigma} \leq r$. $\sigma = \bar{\sigma}$ otherwise.*

Analysis of $\pi^B|_{\lambda < w}$ with $\sigma = 0$

Assume the condition $\phi_\varepsilon \bar{\sigma} \leq r$ is satisfied. Then the profit-maximization problems simplify as follows:

$$\pi^B|_{B \geq wD} = \max_{L,B,D,E} \quad \left[R^L L + R^B B - R^D D - g(D,L) \right] / R^E - E$$
$$\text{s.t.} \quad L + B = E + D, \quad B \geq wD, \quad E \geq \gamma L$$

$$\pi^B|_{liq.\,risk} = \max_{L,B,D,E} \quad p \left[R^L L + R^B B - R^D D - g(D,L) \right] / R^E - E$$
$$\text{s.t.} \quad L + B = E + D, \quad B \geq \lambda D, \quad E \geq \gamma L$$

Recall that $\pi^B|_{\lambda < w} = \max\{\pi^B|_{liq.\,risk}, \pi^B|_{B \geq wD}\}$. I will first analyze $\pi^B|_{B \geq wD}$, then $\pi^B|_{liq.\,risk}$. Due to the isomorphisms between these two problems on the one hand and $\pi^B|_{\lambda \geq w}$ on the other hand, the solutions follow almost immediately from proposition 2.

Solution to $\pi^B|_{B \geq wD}$ with $\sigma = 0$ Recall that the problem of maximizing $\pi^B|_{B \geq wD}$ is the same as the problem of maximizing $\pi^B|_{\lambda \geq w}$ if λ is replaced by w in the latter. Thus, defining $\rho_w \equiv \frac{1-\gamma}{1-w}$, adapting the notation $\tilde{R}^D(w) = R^D + \frac{w}{1-w}(R^D - R^B)$, and referring to the $B \geq wD$ constraint as the 'liquidity constraint', we have

Proposition 7 (Solution to problem $\pi^B|_{B \geq wD}$ with $\sigma = 0$.) *A finite solution requires*

$$R^B \leq R^D + g_D(D,L) \leq R^L - g_L(D,L) \leq R^E \tag{34}$$

The liquidity constraint ($B \geq wD$) binds if and only if the first inequality is strict. The capital requirement binds if and only if the last inequality is strict or, equivalently, if and only if $\tilde{R}^D(w) + \frac{1}{1-w} g_D(D,L) < R^E$. The solution satisfies the zero-profit condition:

$$R^L - g_L(D,L) = \gamma R^E + (1-\gamma)\{\tilde{R}^D(w) + \frac{1}{1-w} g_D(D,L)\} \tag{35}$$

resulting in $\pi^B = 0$. Four cases are possible, which are as described in proposition 2, with ρ_w in place of ρ, w in place of λ, and 'liquidity constraint' in place of 'liquidity requirement'.

Solution to $\pi^B|_{liq.\,risk}$ with $\sigma = 0$ Recall that the problem of maximizing $\pi^B|_{liq.\,risk}$ is the same as the problem of maximizing $\pi^B|_{\lambda \geq w}$ if R^E is replaced by R^E/p in the latter. Hence,

Lemma 2 (Solution to problem $\pi^B|_{liq.\,risk}$ with $\sigma = 0$) *A finite solution requires*

$$R^B \leq R^D + g_D(D, L) \leq R^L - g_L(D, L) \leq R^E/p \tag{36}$$

The liquidity requirement binds if and only if the first inequality is strict. The capital requirement binds if and only if the last inequality is strict or, equivalently, if and only if $\tilde{R}^D(\lambda) + \frac{1}{1-\lambda}g_D(D, L) < R^E/p$. The solution satisfies the zero-profit condition:

$$R^L - g_L(D, L) = \gamma R^E/p + (1 - \gamma)\{\tilde{R}^D(\lambda) + \frac{1}{1 - \lambda}g_D(D, L)\} \tag{37}$$

resulting in $\pi^B = 0$. Four cases are possible, which are as described in proposition 2, with R^E/p in place of R^E.

Note that realized profits are easily derived from zero expected economic profits (that is, $\pi^B|_{liq.\,risk} = prL/R^E - E = 0$). Economic profits conditional on $(\eta = 0)$ are $rL/R^E - E = 0 + (1 - p)rL/R^E = (1 - p)E/p$ so shareholders earn a rate of return $R^E(1 + \frac{1-p}{p}) = R^E/p$ in that event. Economic profits conditional on $(\eta = 1)$ are $-E$ and shareholders lose all their investment in that event.

Solution to $\pi^B|_{\lambda < w}$ with $\sigma = 0$ Recall that

$$\pi^B|_{\lambda < w} = \max\{\pi^B|_{B \geq wD}, \pi^B|_{liq.\,risk}\}$$

Let δ_w^* be an optimal choice for the ratio D/L associated with problem $\pi^B|_{B \geq wD}$ and define δ_{lr}^* analogously for problem $\pi^B|_{liq.\,risk}$. The zero-profit condition (35) for problem $\pi^B|_{B \geq wD}$ provides an expression for the break-even lending rate for this problem that is consistent with optimal choice:

$$R^L|_{B \geq wD}^{break-even} = \gamma R^E + (1 - \gamma)\{\tilde{R}^D(w) + \frac{1}{1 - w}g_D(\delta_w^*, 1)\} + g_L(\delta_w^*, 1)$$

(Recall that the partial derivatives of g are homogenous of degree 0.) Similarly, for $\pi^B|_{liq.\,risk}$, we have

$$R^L|_{liq.\,risk}^{break-even} = \gamma R^E/p + (1 - \gamma)\{\tilde{R}^D(\lambda) + \frac{1}{1 - \lambda}g_D(\delta_{lr}^*, 1)\} + g_L(\delta_{lr}^*, 1)$$

To have a finite solution to $\pi^B|_{\lambda<w}$, it must be the case that

$$R^L = \min\{R^L|^{break-even}_{B\geq wD}, R^L|^{break-even}_{liq.\,risk}\}$$

The reason is as follows: (i) if $R^L < \min\{R^L|^{break-even}_{B\geq wD}, R^L|^{break-even}_{liq.\,risk}\}$, no bank would operate with a strictly positive scale (lest it earns strictly negative profits), a situation that is ruled out by equilibrium conditions; (ii) if $R^L > \min\{R^L|^{break-even}_{B\geq wD}, R^L|^{break-even}_{liq.\,risk}\}$, then the business model with the lowest break-even rate would yield infinite profits by operating at infinite scale, something that is again ruled out by equilibrium conditions. (In all this, recall that g is linear homogenous and all contraints are linear, so each of the problems is linear homogenous in (L, B, D, E).)

Moreover, the business model with the *lowest* break-even lending rate will be operated in equilibrium. That is, provided a finite solution exists, $\pi^B|_{\lambda<w} = \pi^B|_{B\geq wD}$ if $R^L|^{break-even}_{B\geq wD} \leq R^L|^{break-even}_{liq.\,risk}$ and $\pi^B|_{\lambda<w} = \pi^B|_{liq.\,risk}$ otherwise. Now,

$$R^L|^{break-even}_{B\geq wD} - R^L|^{break-even}_{liq.\,risk} = \gamma R^E\left(1 - \frac{1}{p}\right) + (1-\gamma)\left(\frac{w}{1-w} - \frac{\lambda}{1-\lambda}\right)(R^D - R^B) + h$$

where h collects terms related to differences in intermediation costs between the two business models:[43]

$$h \equiv \rho_w g_D(\delta^*_w, 1) + g_L(\delta^*_w, 1) - \rho g_D(\delta^*_{lr}, 1) - g_L(\delta^*_{lr}, 1) \tag{38}$$

Lemma 3 *Suppose $\lambda < w$ and $\phi_\varepsilon \bar{\sigma} \leq r$, so $\sigma = 0$. Let*

$$\zeta \equiv (1-\gamma)\left(\frac{w}{1-w} - \frac{\lambda}{1-\lambda}\right)(R^D - R^B) - \gamma\left(\frac{1-p}{p}\right)R^E + h$$

If $\zeta \leq 0$ and a finite solution to $\pi^B|_{B\geq wD}$ exists, then $\pi^B|_{\lambda<w} = \pi^B|_{B\geq wD}$ and proposition 7 applies; the bank self-insures against liquidity stress. If $\zeta > 0$ and a finite solution to $\pi^B|_{liq.\,risk}$ exists, then $\pi^B|_{\lambda<w} = \pi^B|_{liq.\,risk}$ and lemma 2 applies; the bank is at risk of failure due to liquidity stress.

It is straightforward to show that $\zeta > 0$ implies $R^B < R^D + g_D(D, L)$, so the liquidity requirement binds whenever $\pi^B|_{\lambda<w} = \pi^B|_{liq.\,risk}$. Hence, by imposing $\zeta > 0$ on the solution in lemma 2, we have:

Proposition 8 (Solution to problem $\pi^B|_{liq.\,risk}$ with $\sigma = 0$ and $\zeta > 0$) *A finite solution requires*

$$R^B \leq R^D + g_D(D, L) \leq R^L - g_L(D, L) \leq R^E/p$$

---

[43]If the capital requirement and the liquidity constraint are both binding for each problem, then $h = g(\rho_w, 1) - g(\rho, 1)$ (using Euler's theorem). As $\lambda < w$, $\rho < \rho_w$, so $h \geq 0$ in this case.

With $\zeta > 0$, the first inequality is strict, so the liquidity requirement always binds and $B = \lambda D$. The capital requirement binds if and only if the last inequality is strict or, equivalently, if and only if $\tilde{R}^D(\lambda) + \frac{1}{1-\lambda}g_D(D,L) < R^E/p$. The solution satisfies the zero-profit condition:

$$R^L - g_L(D,L) = \gamma R^E/p + (1-\gamma)\{\tilde{R}^D(\lambda) + (1-\lambda)^{-1}g_D(D,L)\}$$

resulting in $\pi^B = 0$. Two cases are possible:

1. If $R^L - g_L(\rho, 1) \geq R^E/p$, then the capital requirement is slack, and

$$R^L - g_L(D,L) = R^E/p = \tilde{R}^D(\lambda) + (1-\lambda)^{-1}g_D(D,L)$$

2. If $R^L - g_L(\rho, 1) < R^E/p$, then the capital requirement binds, so $E = \gamma L$, and

$$R^L = \gamma R^E/p + (1-\gamma)\tilde{R}^D(\lambda) + g(\rho, 1)$$

Credit risk choice with $\lambda < w$, part 2

Recall that if and only if $\phi_\varepsilon \bar{\sigma} \leq r$ ($\equiv R^L + R^B(B/L) - R^D(D/L) - g(D/L, 1)$), then $\sigma = 0$ and, in that case, $\pi^B|_{B \geq wD} = rL/R^E - E = 0$ and $\pi^B|_{liq.\,risk} = prL/R^E - E = 0$ (see subsection 2.1.2, lemma 1 and propositions 7 and 8). (The fact that in each case the solution entails zero economic profits (that is, $\pi^B = 0$) reflects the fact that each of the problems is linear homogenous in (L, B, D, E), due to constant returns to scale and the linearity of the constraints.) As a result:

- If $\zeta \leq 0$, then $\pi^B|_{\lambda < w} = \pi^B|_{B \geq wD} = 0$ implies $r = R^E(E/L) \geq R^E\gamma$, so $\phi_\varepsilon \bar{\sigma} \leq \gamma R^E$ is a sufficient condition for no excessive risk taking (and necessary if the capital requirement binds).

- If $\zeta > 0$, then $\pi^B|_{\lambda < w} = \pi^B|_{liq.\,risk} = 0$ implies $r = R^E E/(pL) \geq R^E\gamma/p$, so $\phi_\varepsilon \bar{\sigma}p \leq \gamma R^E$ is a sufficient condition for no excessive risk taking (and necessary if the capital requirement binds).

Interestingly, a (slightly) lower level of capital requirement is sufficient to deter excessive risk taking if the optimal choice involves liquidity risk, taking R^E and $\bar{\sigma}$ as given. More importantly, $\phi_\varepsilon \bar{\sigma} \leq \gamma R^E$ is always sufficient, even if $\lambda < w$. Combined with the result in (13), this yields proposition 3 in the main text.

Finally, combining this with lemma 3 and proposition 8 yields proposition 4 in the main text. QED.

Appendix B. Constrained social planner's problem

Equivalence to competitive equilibrium

The Lagrangian and first-order conditions to the planner's problem in (27) are:

$$\mathcal{L} = \max_{\{c_t, d_t, b_t, L_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{ u(c_t, d_t, b_t) + \omega_t^{sp}[F(K_t, 1) + (1-\delta)K_t - c_t - K_{t+1} - g(d_t, L_t) - T]$$

$$+ \Lambda_t^{sp}[\bar{B} - b_t - \lambda d_t] + \chi_t^{sp}[(1-\gamma)L_t + \bar{B} - b_t - d_t] + \mu_t^{sp}[K_t - L_t]$$

$$
\begin{aligned}
(c) \quad u_c(c_t, d_t, b_t) &= \omega_t^{sp} \\
(d) \quad u_d(c_t, d_t, b_t) &= \omega_t^{sp} g_D(d_t, L_t) + \Lambda_t^{sp}\lambda + \chi_t^{sp} \\
(b) \quad u_b(c_t, d_t, b_t) &= \Lambda_t^{sp} + \chi_t^{sp} \\
(L) \quad \chi_t^{sp}(1-\gamma) &= \omega_t^{sp} g_L(d_t, L_t) + \mu_t^{sp} \\
(K) \quad \omega_t^{sp}[F_K(K_t, 1) + 1 - \delta] &= \beta^{-1}\omega_{t-1}^{sp} - \mu_t^{sp}
\end{aligned}
$$

with $\Lambda_t^{sp} \geq 0$, $\Lambda_t^{sp}[\bar{B} - b_t - \lambda d_t] = 0$, $\chi_t^{sp} \geq 0$, $\chi_t^{sp}[(1-\gamma)L_t + \bar{B} - b_t - d_t] = 0$, $\mu_t^{sp} \geq 0$, and $\mu_t^{sp}[K_t - L_t] = 0$.

Subtract λ times the first-order condition with respect to bonds (FOC(b)) from FOC(d) to obtain $u_d - \lambda u_b = (1-\lambda)\chi_t^{sp} - \omega_t^{sp}g_D$ (omitting arguments for brevity). Solving for χ_t^{sp} and inserting the result into FOC (L) and using FOC(c) yields:

$$\frac{\mu_t^{sp}}{\omega_t^{sp}} = \rho\left(\frac{u_d(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} - \lambda\frac{u_b(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} - g_D(d_t, L_t)\right) - g_L(d_t, L_t) \equiv \Delta_K(c_t, d_t, b_t, L_t)$$

(Recall that $\rho = (1-\gamma)/(1-\lambda)$.) Inserting this into FOC(K) and again using FOC(c) yields:

$$F_K(K_t, 1) + 1 - \delta = \beta^{-1}\frac{u_c(c_{t-1}, d_{t-1}, b_{t-1})}{u_c(c_t, d_t, b_t)} - \Delta_K(c_t, b_t, d_t, L_t)$$

This replicates equations (20), (21) and (22) in the characterization of the decentralized equilibrium. Furthermore, (23) follows from $\Delta_K(c_t, d_t, b_t, L_t) = \mu_t^{sp}/\omega_t^{sp}$, $\omega_t^{sp} = u_c > 0$, $\mu_t^{sp} \geq 0$, and $\mu_t^{sp}[K_t - L_t] = 0$.

Since $\mu_t^{sp} + \omega_t^{sp}g_L = \chi_t^{sp}(1-\gamma)$ (from FOC(L)), $\mu_t^{sp} \geq 0$, $\omega_t^{sp} > 0$, $g_L \geq 0$, $\chi_t^{sp} \geq 0$ and $\chi_t^{sp}[(1-\gamma)L_t + \bar{B} - b_t - d_t] = 0$, it follows that $\chi_t^{sp} > 0$ and $d_t = (1-\gamma)L_t + \bar{B} - b_t$ if $\mu_t^{sp} > 0$ or if $g_L > 0$ (or both); otherwise $\chi_t^{sp} = 0$ and $d_t \leq (1-\gamma)L_t + \bar{B} - b_t$, a result that is equivalent to (24) in the decentralized equilibrium.

Taking the difference between FOC(b) and FOC(d) yields

$$(1-\lambda)\frac{\Lambda_t^{sp}}{\omega_t^{sp}} = \frac{u_b(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} - \frac{u_d(c_t, d_t, b_t)}{u_c(c_t, d_t, b_t)} + g_D(d_t, L_t) \equiv \Delta_B(c_t, d_t, b_t, L_t) \qquad (39)$$

the expression in (26). (25) follows from $\Delta_B(c_t, d_t, b_t, L_t) = (1-\lambda)\Lambda_t^{sp}/\omega_t^{sp}$, $\omega_t^{sp} = u_c > 0$, $\Lambda_t^{sp} \geq 0$, and $\Lambda_t^{sp}[\bar{B} - b_t - \lambda d_t] = 0$.

Finally, equation (19) in the characterization of the decentralized equilibrium is included as one of the constraints of the planner's problem. Collecting these results, it is apparent that the allocations of K_t, c_t, b_t, d_t and L_t implied by the planner's problem are identical to those of the decentralized equilibrium summarized in equations (19)-(26). Hence, the constrained social planner's problem replicates the decentralized equilibrium if $\lambda \geq w$ and (14) holds for all $t \geq 0$ in that equilibrium, so that $\sigma_t = 0$. Moreover, under those conditions, welfare equals $V_0(\theta)$, as defined in (27).

Proof of proposition 5

Call the current period 0. Using the envelope theorem, the marginal effect on welfare of raising the liquidity requirement λ is:

$$
\begin{aligned}
\frac{\partial V_0(\theta)}{\partial \lambda} &= -\sum_{t=0}^{\infty} \beta^t \Lambda_t^{sp} d_t \\
&= -\sum_{t=0}^{\infty} \beta^t \{u_b(c_t, d_t, b_t) - u_d(c_t, d_t, b_t) + u_c(c_t, d_t, b_t)g_D(d_t, L_t)\}\frac{d_t}{1-\lambda}
\end{aligned}
$$

(see (39)). Since the allocations of c_t, d_t, b_t and L_t are identical to those of the decentralized equilibrium, their equilibrium values can be used. Moreover, in that equilibrium, we have, by taking the difference between the household's first-order conditions (2) and (3),

$$
u_b(c_t, d_t, b_t) - u_d(c_t, d_t, b_t) = u_c(c_t, d_t, b_t)(R_t^D - R_t^B)
$$

Thus, with the assumption that the economy is in steady state in period 0,

$$
\frac{\partial V_0(\theta)}{\partial \lambda} = -\frac{u_c(c_0, d_0, b_0)(R_0^D - R_0^B + g_D(d_0, L_0))d_0}{(1-\beta)(1-\lambda)}
$$

As is standard, compare this to the welfare effect of a permanent change in consumption by a factor $(1+\nu)$, which equals, to a first-order approximation, $\sum_{t=0}^{\infty} \beta^t u_c(c_t, d_t, b_t)c_t\nu$, or $u_c(c_0, d_0, b_0)c_0\nu/(1-\beta)$ with a steady state reigning in period 0. Equating this to the right-hand side of the previous equation yields proposition 5. QED.

Proof of proposition 6

Call the current period 0. Using the envelope theorem, the marginal effect on welfare of raising γ is:

$$
\begin{aligned}
\frac{\partial V_0(\theta)}{\partial \gamma} &= -\sum_{t=0}^{\infty} \beta^t \chi_t^{sp} L_t \\
&= -\sum_{t=0}^{\infty} \beta^t \{u_d(c_t, d_t, b_t) - \lambda \, u_b(c_t, d_t, b_t) - u_c(c_t, d_t, b_t)g_D(d_t, L_t)\}\frac{L_t}{1-\lambda}
\end{aligned}
$$

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where the second equality follows from the planner's first-order conditions for bonds, deposits and consumption. Since the allocations of c_t, d_t, b_t and L_t are identical to those of the decentralized equilibrium, their equilibrium values can be used. Moreover, in that equilibrium, we have, from the household's first-order conditions (2) and (3),

$$
\begin{aligned}
u_d(c_t, d_t, b_t) - \lambda u_b(c_t, d_t, b_t) &= u_c(c_t, d_t, b_t)(R_t^E - R_t^D - \lambda(R_t^E - R_t^B)) \\
&= u_c(c_t, d_t, b_t)(1 - \lambda)(R_t^E - \tilde{R}_t^D(\lambda))
\end{aligned}
$$

Hence,

$$
\frac{\partial V_0(\theta)}{\partial \gamma} = -\sum\nolimits_{t=0}^{\infty} \beta^t u_c(c_t, d_t, b_t)(R_t^E - \tilde{R}_t^D(\lambda) - \frac{g_D(d_t, L_t)}{1 - \lambda})L_t
$$

With the assumption that the economy is in steady state in period 0,

$$
\frac{\partial V_0(\theta)}{\partial \gamma} = -(1 - \beta)^{-1} u_c(c_0, d_0, b_0)(R_0^E - \tilde{R}_0^D(\lambda) - \frac{g_D(d_t, L_t)}{1 - \lambda})L_0
$$

Compare this to the welfare effect of a permanent change in consumption by a factor $(1+\nu)$, which equals, to a first-order approximation, $\sum_{t=0}^{\infty} \beta^t u_c(c_t, d_t, b_t)c_t\nu$, or $u_c(c_0, d_0, b_0)c_0\nu/(1-\beta)$ with a steady state in period 0. Equating this to the right-hand side of the previous equation yields the proposition. QED.

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