

MONEY RUNS*

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Abstract

We present a banking model in which bank debt circulates in decentralized secondary markets, like banknotes did in the nineteenth century and repos do today. We find that bank debt is susceptible to runs because secondary-market liquidity is subject to sudden, self-fulfilling dry-ups. When debt fails to circulate it is redeemed on demand in a “money run.” Even though demandable debt exposes banks to costly runs, banks still choose to issue it: the option to redeem on demand increases secondary-market debt prices and hence increases primary-market debt capacity—i.e. demandability and tradeability are complements, unlike in existing models.

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1 Introduction

Bank debt was a major form of money in the nineteenth century United States. To get beer from the barman, you would exchange banknotes over the counter. Banknotes were redeemable on demand and sudden redemptions—bank runs—were common. Bank debt remains a major form of money today. To get liquidity from a financial counterparty, you exchange repos in an over-the-counter (OTC) market. Repos are effectively redeemable on demand and sudden redemptions—repo runs—were a salient event of the 2008–2009 financial crisis.^{1,2} In summary, when you hold bank debt, you can get liquidity either by trading OTC or, alternatively, by demanding redemption from the issuing bank. But demanding redemption comes with the risk of a run. Why would you run on a bank rather than trade its debt in the market? In other words, why is bank debt susceptible to costly runs, even though it is tradeable? Moreover, why do banks choose to borrow via demandable debt, even though it exposes them to costly runs?

To give a new perspective on these questions, we focus on how banks create money by issuing liabilities that circulate in OTC markets, like banknotes did in the nineteenth-century and repos do today. In the model, bank debt is susceptible to runs because liquidity in the OTC market is fragile, and subject to sudden, self-fulfilling dry-ups.

¹Gorton and Metrick (2009, 2012) and Krishnamurthy, Nagel, and Orlov (2014) provide descriptions of repo runs.

²Gorton (2012b) argues that it remains a theoretical challenge to understand how these runs arise and how they affect the design of bank liabilities that circulate as money. He suggests that, because banknotes and repos are backed by collateral, there is no “common pool problem” inducing depositors to race to withdraw first in the event of a crisis:

In the U.S. under state free banking laws banks were required to back their notes with state bonds. In the case of a bank failure—an inability to honor requests for cash from noteholders—the state bonds would be sold (by the state government) and the note holders paid off pro rata. Note holders were paid off pro rata, so there was no common pool problem. Yet, there was a run on banks (banknotes and deposits) during the Panic of 1857 (p. 15).

And he goes on to say:

Generating such [a run] event in a model seems harder when...the form of money [is such that] each “depositor” receives a bond as collateral. There is no common pool of assets on which bank debt holders have a claim. So, strategic considerations about coordinating with other agents do not arise. This is a challenge for theory and raises issues concerning notions of liquidity and collateral, and generally of the design of trading securities—private money (p. 2).

We generate such runs on bank debt in a model in which banks optimally design securities that circulate in secondary markets. These runs occur because strategic considerations about coordinating with other agents do arise in the secondary market. (Other papers, such as Martin, Skeie, and von Thadden (2014a, 2014b) and Kuong (2015) study other mechanisms by which runs on backed debt can occur.)

When debt fails to circulate it is redeemed on demand in a bank run, or “money run.” Such runs were common in the nineteenth century US, when depositors ran on banks after “the bank note that passed freely yesterday was rejected this morning” (Treasury Secretary Howell Cobb (1858), quoted in Gorton (2012a) p. 36). Even though demandable debt exposes banks to costly runs, banks still choose to issue it. In our model, this is because it increases banks’ debt capacity: the option to redeem on demand props up the price of debt in the *secondary market*. In other words, we find that demandability and tradeability are complements. This contrasts with Jacklin’s (1987) argument that, roughly, you do not need the option to redeem debt on demand if you can just trade it in the secondary market. In our model, on the contrary, you do: the option to redeem debt on demand increases the price at which you trade it in the secondary market. This benefits the issuing bank by allowing it to borrow more. As a result, financial fragility is a necessary evil—it is the cost of the debt capacity afforded by demandable debt. Overall, our model exposes a new type of run and a new rationale for demandable debt, both of which are based on the circulation of bank liabilities in the secondary market.

Model preview. In the model, a borrower B has an investment opportunity and needs to borrow from a creditor C_0 to fund it. The model is based on two key assumptions. First, there is a horizon mismatch, similar to that in Diamond and Dybvig (1983): C_0 may be hit by a liquidity shock before B’s investment pays off. This mismatch implies that B does maturity transformation, and therefore resembles a bank. Second, B’s debt is traded in an OTC market, similar to that in Trejos and Wright (1995) and Duffie, Gârleanu, and Pedersen (2005): if C_0 is hit by a liquidity shock before B’s investment pays off, C_0 can match with a counterparty C_1 and bargain bilaterally to trade B’s debt. Likewise, C_1 may be hit by a liquidity shock before B’s investment pays off, in which case it can match with a counterparty C_2 and bargain bilaterally to trade B’s debt, and so on. If B’s debt is demandable, then a creditor may redeem it before the investment pays off, forcing B to liquidate inefficiently.

Results preview. Our first main result is that B’s debt capacity is highest if it issues tradeable, demandable debt, which we refer to as a “banknote.” In particular, as long as the horizon mismatch is sufficiently severe, B cannot fund its investment with non-tradeable debt (e.g. a bank loan), even if it is demandable, or with non-demandable debt (e.g. a bond), even if it is tradeable. To see why this is, consider C_0 ’s decision whether or not to lend to B. C_0 knows that he may be hit by a liquidity shock before B’s investment pays off, in which case C_0 liquidates B’s debt, either by redeeming on demand or by trading in the OTC market. If B’s debt is not tradeable (but is demandable), then C_0 must redeem on demand, forcing B into inefficient liquidation and recovering less than his initial investment. If the horizon mismatch is severe, then this loss from early redemption is so likely that C_0 is unwilling to lend in the first place.

In contrast, if B’s debt is tradeable (but is not demandable), then C_0 can avoid early redemption by trading with C_1 in the OTC market. However, C_0 ’s liquidity shock puts him in a weak bargaining position with C_1 : C_0 has a low outside option because he has no way to get liquidity if trade fails. As a result, he sells B’s debt at a discounted price, recovering less than his initial investment. If the horizon mismatch is severe, then this loss from selling at a discount is so likely that C_0 is unwilling to lend in the first place. But if B’s debt is demandable as well as tradeable, then debt does not trade at such a high discount in the secondary market. This is because demandability improves C_0 ’s bargaining position with C_1 . It increases his outside option, since he can redeem on demand when trade fails. As a result, C_0 can trade B’s debt at a high price following a liquidity shock. Thus, C_0 is insured against liquidity shocks, making him willing to fund B’s investment. In contrast to existing models of demandable debt, our model suggests that demandability complements tradeability—just the option to redeem on demand props up the resale price of debt in secondary markets even if the debt is never actually redeemed on demand in any state of the world.

Our second main result is that banknotes are susceptible to a new kind of bank run, which results directly from the dry-up of secondary-market liquidity. Specifically, a sudden (but rational) change in beliefs can cause counterparties to stop trading in the secondary market, leading the creditor to redeem on demand and forcing B into inefficient liquidation. Observe that this run occurs even though B has only a single creditor—there is no static coordination problem in which multiple creditors race to be the first to withdraw from an issuing bank as in Diamond and Dybvig (1983); rather, there is a dynamic coordination problem in the secondary market in which a counterparty does not accept B’s debt today because he is worried that his future counterparty will not accept B’s debt tomorrow. Due to this self-fulfilling liquidity dry-up, B’s creditor is suddenly unable to trade when he is hit by a liquidity shock and, thus, he must demand redemption from B. We refer to this run as a “money run” because it is the result of the failure of B’s debt to function as a liquid money in the secondary market.

Finally, we show that if the horizon mismatch is not too severe, B may choose to borrow via a bond, i.e. tradeable, non-demandable debt, instead of a banknote. The reason is not only that bond liquidity dry-ups do not lead to bank runs and inefficient liquidation, but also that bond liquidity dry-ups are relatively “unlikely.” This is because bonds trade at a larger discount than banknotes, making counterparties more willing to trade in the secondary market (Subsection 3.8).

Policy. Our analysis suggests that financial fragility may be a necessary evil given secondary-market trading frictions—money runs are the cost of the debt capacity afforded by demandable debt. However, decreasing secondary-market trading frictions can make banks less reliant on demandable debt, decreasing the likelihood of runs.

Thus, we suggest that to improve bank stability, a policy maker may be better off decreasing secondary-market frictions than regulating banks directly. In contemporary markets, we think that centralized exchanges and clearing houses for bank bonds, like those for stocks, could decrease trading frictions in a way that decreases banks' reliance on overnight (effectively demandable) debt.

Unlike in Diamond and Dybvig's (1983) model of bank runs, in which suspension of convertibility restores efficiency, in our model suspension of convertibility may have an adverse effect. Since it prevents creditors from redeeming on demand to meet their liquidity needs, it leads to lower secondary-market debt prices and, hence, to constrained bank borrowing and inefficient investment.

Empirical content. As mentioned above, our model is motivated by empirical observations about financial fragility and circulating bank debt, such as banknotes and repos. In particular, our model offers an explanation of the following facts: (i) runs on bank debt are relatively common, even when the debt is backed by collateral; (ii) runs are often precipitated by the failure of debt to circulate in secondary markets; and (iii) banks choose to borrow via demandable debt even though it exposes them to costly runs.

Our model also casts light on several other stylized facts. (i) Demandable bank instruments, such as banknotes, deposits, and repos, are more likely to serve as media of exchange than other negotiable instruments, such as bonds and shares. In the model, this is because the option to redeem on demand props up the secondary-market price of bank debt. Thus, if you hold a variety of instruments, you prefer to use demandable instruments to raise liquidity in the secondary market and to hold long-term instruments till maturity. (ii) Our model casts light on why bank debt is more likely to be demandable than corporate debt. Banks, almost by definition, have a horizon mismatch between their assets and liabilities—they perform maturity transformation. In the model, this horizon mismatch prevents you from borrowing via other instruments. Corporates are less likely to suffer from the horizon mismatch, and are therefore more likely to fund themselves with bonds or bank loans, which do not expose them to costly runs/liquidation. (iii) Our model casts light on why nineteenth-century banknotes traded at a greater discount in markets farther away from the issuing bank: distance from the issuer made the notes harder to redeem on demand, weakening note holders' bargaining positions in the secondary market and decreasing the price of banknotes (see Gorton (1996)). (iv) Our model generates runs even with a single depositor, consistent with the fact that many runs are not market-wide, but rather occur in isolation. Indeed, Krishnamurthy, Nagel, and Orlov (2014) find that repo runs occurred in relative isolation during the financial crisis.

Application to repos. Formally, repos are collateralized bilateral contracts, not

circulating negotiable instruments like the banknotes in our model. However, repos share the key features of our banknotes: in addition to being exchanged OTC, they are effectively demandable and tradeable. They are effectively demandable because they are continuously rolled over (not settled and reopened daily). Thus, not rolling over a repo position is effectively redeeming on demand. They are effectively tradeable because repo collateral is constantly rehypothecated³ (see Singh and Aitken (2010) and Singh (2010)). Thus, repo creditors use repos to get liquidity from third parties in the event of a liquidity shock, as creditors use tradeable debt to get liquidity in our model.

Related literature. We make three main contributions to the literature.

First, we offer a new rationale for demandable debt. This adds to the literature in two ways. (i) It complements the literature that shows how demandability can mitigate moral hazard problems (Calomiris and Kahn (1991) and Diamond and Rajan (2001a, 2001b)).⁴ In particular, we show how demandability can facilitate secondary-market trade in “private money.” Thus, our model connects two of the main features of bank liabilities: they both circulate as money and are redeemable on demand. (ii) It provides a counterpoint to the literature that suggests that tradeability can substitute for demandability. Notably, Jacklin (1987) shows that, in Diamond and Dybvig’s (1983) environment, you do not need to redeem debt on demand if you can just trade it in the secondary market.⁵ We show that if bank debt is traded in an OTC market, like banknotes, deposits, and repos are, then demandability serves another purpose: it improves sellers’ bargaining positions and thus increases the secondary-market price of bank debt. Thus, demandability complements tradeability in our model.

Second, we uncover a new kind of bank run. By connecting the fragility of money to the fragility of banks, this adds both to the literature on coordination-based bank-run models following Diamond and Dybvig (1983) and to the literature on search-based money models following Kiyotaki and Wright (1989, 1993). In these money models, monetary exchange is fragile since trade is self-fulfilling. Similarly, in the bank run

³So whereas the repo contract itself technically does not circulate, the repo collateral does. See Lee (2015) for a model focusing on collateral circulation.

⁴In their conclusion, Diamond and Rajan (2001a) make the link between demandability and circulating bank notes informally, saying that

deposits are readily transferable, and liquid, because buyers of deposits have no less ability to extract payment than do sellers of deposits. Thus, the deposits can serve as bank notes or checks that circulate between depositors. This could explain the special role of banks in creating inside money (p. 425).

We make this link formally in this paper.

⁵Other papers show that there may still be a role for demandability if tradeability is limited (Allen and Gale (2004), Antinolfi and Prasad (2008), Diamond (1997), and von Thadden (1999)). In these models, banks issue demandable debt *in spite of* trade in secondary markets, e.g. to overcome trading frictions, such as limited market participation. In our model, banks issue demandable debt *because of* trade in secondary markets—the option to redeem on demand improves the terms of trade in the secondary market.

models, bank deposits are fragile since withdrawals are self-fulfilling. To the best of our knowledge, we are the first to show that such bank fragility follows immediately from such monetary fragility⁶ and, hence, coordination-based bank runs can occur even with a single depositor—i.e. without multiple depositors racing to withdraw from a common pool of assets as in Diamond and Dybvig (1983).⁷ This helps to explain how runs can occur on collateral-backed debt, complementing the existing literature (see footnote 2).

Third, we study security design when securities are traded OTC in the secondary market. This adds to the literature in three ways. (i) It complements the search-based money literature which analyzes which type of asset is the socially optimal medium of exchange for trade in the secondary market (e.g., Kiyotaki and Wright (1989) and Burdett, Trejos, and Wright (2001)). We analyze which type of contract is the privately optimal circulating instrument for funding in the primary market.⁸ (ii) It extends results in the literature on corporate bonds that suggests short-maturity bonds may have high resale prices in the secondary market (Bruche and Segura (2016) and He and Milbradt (2014)). These papers restrict attention to debt contracts as in Leland and Toft (1996). We point out that with more general contracts the option to redeem on demand provides a way to prop up secondary-market prices that does not require rolling over maturing debt.⁹ (iii) It provides a counterpoint to the literature that suggests that security design may prevent bank runs (e.g. Andolfatto, Nosal, and Sultanum (forthcoming), Green and Lin (2003), and Peck and Shell (2003)). This literature suggests that if the space of securities is rich enough, then bank runs do not arise in Diamond and Dybvig’s (1983) environment. Our analysis suggests that the security designs proposed in this literature may not prevent all kinds of bank runs. This is because, in our environment, it is exactly the possibility of a run, i.e. the option to redeem on demand, that props up the secondary-market price of the banknote and thus

⁶A number of papers study bank money creation independently of financial fragility (e.g., Donaldson, Piacentino, and Thakor (2016), Gu, Mattesini, Monnet, and Wright (2013), and Kiyotaki and Moore (2001)) and some others embed Diamond–Dybvig runs in economies with private money (e.g., Champ, Smith, and Williamson (1996) and Sanches (2015)). Relatedly, Sanches (2016) argues that banks’ inability to commit to redeem deposits can make private money unstable.

⁷Our focus on runs that result from dynamic coordination failures among counterparties in the secondary market complements models that focus on runs that result from dynamic coordination failures among depositors in the primary market (the dynamic analog of Diamond–Dybvig-type runs), such as He and Xiong (2012).

⁸A related line of research focuses on information, rather than search frictions, in secondary-market trade (Gorton and Pennacchi (1990), Dang, Gorton, and Hölmstrom (2015a, 2015b), Dang, Gorton, Hölmstrom, and Ordoñez (forthcoming), Gorton and Ordoñez (2014), Jacklin (1989), and Vanasco (2016)). This literature suggests that secondary-market information frictions lead banks to do risk transformation; analogously, we suggest that secondary-market search frictions lead banks to do liquidity transformation.

⁹In an extension, Bruche and Segura (2016) do consider a version of puttable debt. However, they effectively assume it is not tradeable, which shuts down the interaction of demandability and tradeability that is critical to our results.

makes it the optimal funding instrument.

Layout. In Section 2, we present the model. In Section 3, we present the main results. Section 4 is the conclusion. The Appendix contains all proofs and a table of notations.

2 Model

In this section, we present the model.

2.1 Players, Dates, and Technologies

There is a single good, which is the input of production, the output of production, and the consumption good. Time is discrete and the horizon is infinite, $t \in \{0, 1, \dots\}$.

There are two types of players, a single penniless borrower B and infinitely many deep-pocketed creditors C_0, C_1, \dots . Everyone is risk-neutral and there is no discounting. B is penniless but has a positive-NPV investment. The investment costs c at Date 0 and pays off $y > c$ at a random time in the future, which arrives with intensity ρ . Thus, the investment has NPV = $y - c > 0$ and expected horizon $1/\rho$. B may also liquidate the investment before it pays off; the liquidation value is $\ell < c/2$.

B can fund its project by borrowing from a creditor. However, there is a horizon mismatch similar to that in Diamond and Dybvig (1983): creditors may need to consume before B's investment pays off. Specifically, creditors consume only if they suffer "liquidity shocks," which arrive at independent random times with intensity θ (after which they die). In other words, a creditor's expected "liquidity horizon" is $1/\theta$.

2.2 Borrowing Instruments

At Date 0, B borrows the investment cost c from its initial creditor C_0 and negotiates a repayment $R \leq y$ to make when the investment pays off. We refer to this promised repayment as B's "debt." (However, it can also represent an equity claim; debt and equity have equivalent payoffs, since the terminal payoff y is deterministic.)

In addition to the repayment R , two other characteristics define B's debt: *tradeability* and *demandability*. If B's debt is non-tradeable, then B must repay C_0 . In contrast, if B's debt is tradeable, creditors can exchange the debt among themselves—the debtholder at Date t , denoted H_t , may not be the initial creditor C_0 —and B must repay whichever creditor H_t holds its debt. If B's debt is not demandable, i.e. it is "long-term," then B repays only when its investment pays off. In contrast, if B's debt is demandable, then the debtholder can demand repayment at any date. In this case, if

B’s investment has not paid off, B liquidates its investment and repays the liquidation value ℓ .

In summary, B borrows via one of four types of debt instrument: (i) non-tradeable long-term debt, which we refer to as a “loan”; (ii) non-tradeable demandable debt, which we refer to as a “puttable loan”; (iii) tradeable long-term debt, which we refer to as a “bond”; or (iv) tradeable demandable debt, which we refer to as a “banknote,” although it also resembles a bank deposit or a repo. These instruments are summarized in Figure 1. These instruments are effectively all of the feasible Markovian instruments, i.e. contracts that can depend on the state of B’s investment at Date t , but not on the date itself, and do not violate B’s limited-liability constraint.¹⁰

We let v_t denote the Date- t value of B’s debt to a creditor not hit by a liquidity shock.

FIGURE 1: DEBT INSTRUMENTS

	not demandable	demandable
non-tradeable	“loan”	“puttable loan”
tradeable	“bond”	“banknote” (deposits or repos)

2.3 Secondary Debt Market: Search, Bargaining, and Settlement

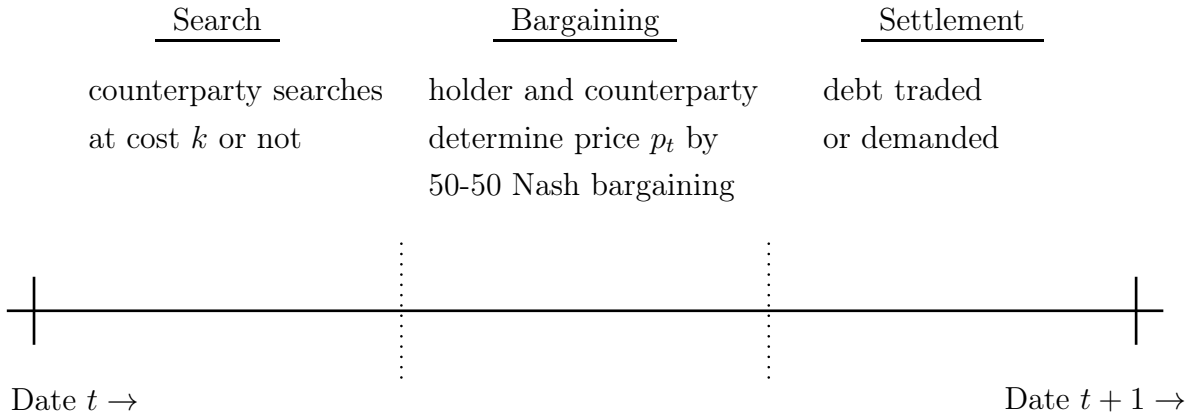
If B has borrowed via tradeable debt, then creditors can trade it bilaterally in an OTC market. At each Date t , C_t is the (potential) counterparty with whom the debtholder H_t may trade B’s debt. C_t can pay a search cost¹¹ k to be matched with the debtholder

¹⁰Here we are making two implicit assumptions that may be worth highlighting. (i) We have implicitly ruled out contracts that depend on creditors’ liquidity shocks. This is an important assumption to generate a reason for a secondary market—if liquidity shocks were contractable, then there would be no role for retrade since the first best could be implemented with contingent contracts à la Arrow–Debreu. In other words, we assume that the Markov state is the state of B’s investment only. (ii) We have implicitly ruled out contracts in which B promises a repayment less than the liquidation value ℓ in the event that debt is demanded early. This is just for simplicity—it helps to streamline the analysis but does not substantively affect the equilibrium.

¹¹This “search” cost can also be interpreted as a cost of trading/transacting/participating in the market, as is common in the finance literature, or as a cost of posting a vacancy, as is common in the labor literature. Further, our main results also obtain if this search cost is zero but the debtholder must pay a per-period holding cost δ as in Duffie, Gârleanu, and Pedersen (2005). Indeed, setting $\delta = (\rho + (1 - \rho)\theta)k$ gives the same ex ante surplus to H_t and C_t . To see this, compute the expected holding cost of the counterparty C_t . If he becomes the debtholder, he holds the debt as long as B’s project does not pay off and he is not hit by a liquidity shock. Thus, at a given date he continues to hold B’s debt with probability $1 - (1 - \rho)(1 - \theta)$. From the formula for the expectation of a geometric distribution, the expected holding cost is thus $[1 - (1 - \rho)(1 - \theta)]^{-1}\delta$. Setting this equal to k gives the expression above. Although the total surplus is the same in the two models, the division of surplus is indeed different, since the cost k is sunk at the time of bargaining.

H_t . If matched, C_t and H_t Nash bargain to determine the price p_t (all Nash bargaining is symmetric).¹² If they agree on a price, then trade is settled: C_t becomes the debtholder in exchange for p_t units of the good. Otherwise, H_t retains the debt. If the debt is demandable, H_t can demand redemption from B or he can remain the debtholder at Date $t + 1$. This sequence of search, bargaining, and settlement is illustrated in Figure 2.

FIGURE 2: SECONDARY-MARKET TRADE



We let σ_t denote C_t 's mixed strategy if H_t is hit by a liquidity shock, so $\sigma_t = 1$ means that C_t searches for sure and $\sigma_t = 0$ means that C_t does not search. Thus, σ_t also represents the probability that H_t finds a counterparty when hit by a liquidity shock. Observe that we restrict attention to C_t 's strategy given H_t is hit by a liquidity shock without loss of generality.¹³

2.4 Timeline

First, B chooses a debt instrument—a loan, a puttable loan, a bond, or a banknote, as described in Subsection 2.2 above. Next, B and the initial creditor C_0 negotiate the repayment R or fail to reach an agreement. If B and C_0 agree on R , then C_0 becomes the initial debtholder. Depending on the instrument, the debtholder may redeem on

¹²To economize on notation, we assume that all bargaining is “symmetric” or “fifty-fifty” Nash bargaining as in Nash (1950). With appropriate modifications of conditions, our results also hold if we use the generalized Nash bargaining outcome, in which H_t and C_t split the surplus in arbitrary fixed proportions. It is important, however, that H_t does not have all the bargaining power, since, in this case, his outside option does not affect the division of surplus. This undoes the effect of demandability.

¹³The reason that this is without loss of generality is that C_t would never search for H_t if he were not hit by a liquidity shock: if H_t is not hit by a liquidity shock, H_t and C_t are identical and there are no gains from trade, so it is never worth it to pay the search cost k for the opportunity to trade.

demand or may trade in the secondary market, as described in Subsection 2.3 above. Formally, the extensive form is as follows.

<u>Date 0</u>	<p>B chooses a debt instrument</p> <p>B is matched with C_0; B and C_0 Nash bargain to determine the repayment R</p> <p>If B and C_0 agree on R, then B invests c. C_0 is the initial debtholder, $H_1 = C_0$</p>
<u>Date $t > 0$</u>	<p><u>If B's investment pays off</u></p> <p style="padding-left: 20px;">B repays R to H_t and B consumes $y - R$</p> <p><u>If B's investment does not pay off: C_t and H_t search, bargain, and settle as described in Subsection 2.3</u></p> <p style="padding-left: 20px;">If there is trade, C_t becomes the new debtholder, $H_{t+1} = C_t$</p> <p style="padding-left: 20px;">If there is no trade, H_t either holds the debt, $H_{t+1} = H_t$, or redeems on demand, in which case B repays ℓ to H_t and B consumes zero</p>

2.5 Equilibrium

The solution concept is subgame perfect equilibrium. An equilibrium constitutes (i) the instrument and associated repayment R , (ii) the price of debt in the secondary market p_t at each date, and (iii) the search strategy σ_t of the potential counterparty C_t ¹⁴ such that B's choice of instrument and C_t 's choice to search are sequentially rational, R and p_t are determined by Nash bargaining, and each player's beliefs are consistent with other players' strategies and the outcomes of Nash bargaining.

We focus on pure, stationary equilibria, i.e. $\sigma_t \equiv \sigma \in \{0, 1\}$ and $p_t \equiv p$.

2.6 Assumption: Horizon Mismatch

We assume that parameters are such that there is a relatively severe horizon mismatch between B's investment and creditors' liquidity needs, as in Diamond and Dybvig (1983). In our infinite-horizon environment, this implies that the expected investment horizon $1/\rho$ must be sufficiently large relative to the expected liquidity horizon $1/\theta$. Specifically, we assume that the following condition holds:

$$\frac{1}{\rho} > \frac{1}{\theta} \frac{2(y-c)}{c(1-\rho)}. \quad (\star)$$

¹⁴Formally, we could also include the debtholder's choice whether to redeem on demand. We omit this, however, and just assume that the debtholder redeems on demand whenever he is hit by a liquidity shock and does not trade the debt. This assumption is just for simplicity; it does not affect the equilibrium.

This assumption implies that B intermediates between short-horizon creditors and a long-horizon investment. This implies B resembles a bank, since, almost by definition, banks transform maturity to meet the needs of short-term depositors and long-term borrowers.

This assumption helps to present our most interesting/novel results in a simple way. We do not need it to characterize the equilibrium. Indeed, we relax it in Subsection 3.8.

3 Results

In this section, we present our main results.

3.1 Borrowing Constraint

We begin the analysis with a necessary condition for B to be able to borrow enough to fund its investment.

LEMMA 1. *For a given instrument, B can borrow and invest if and only if the Date-0 value of its debt exceeds the cost of investment, i.e. if and only if*

$$v_0 \geq c \tag{$}$$

for some repayment $R \leq y$.

We make use of this borrowing constraint repeatedly below when we consider each type of debt instrument in turn and ask whether B can borrow enough to undertake its investment.

3.2 Loan

First, we consider a loan, i.e. non-tradeable long-term debt. At Date t , the value v_t of the loan can be written recursively:

$$v_t = \rho R + (1 - \rho)(1 - \theta)v_{t+1}. \tag{1}$$

The terms are determined as follows. With probability ρ , B's investment pays off and B repays R . With probability $(1 - \rho)\theta$, B's investment does not payoff and the debtholder H_t is hit by a liquidity shock. Since the loan is neither tradeable nor demandable, H_t gets zero. With probability $(1 - \rho)(1 - \theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains B's debt at Date $t + 1$, which has value v_{t+1}

at Date t since there is no discounting.¹⁵ By stationarity ($v_t = v_{t+1} \equiv v$), equation (1) gives

$$v = \frac{\rho R}{\rho + (1 - \rho)\theta}. \quad (2)$$

Given the horizon mismatch assumption (\star), the expression above is always less than the cost of investment c . Thus, if B borrows via a loan, B cannot satisfy its borrowing constraint ($\$$) and therefore cannot invest.

LEMMA 2. *If B borrows via a loan, it cannot fund its investment.*

B cannot borrow from C_0 via a loan even though its project has positive NPV. This is because, given the horizon mismatch, C_0 is likely to be hit by a liquidity shock and need to consume before the project pays off, in which case C_0 gets zero, since the loan is neither tradeable nor demandable.

3.3 Puttable Loan

Now we consider a puttable loan, i.e. non-tradeable demandable debt. At Date t , the value v_t of the puttable loan can be written recursively:

$$v_t = \rho R + (1 - \rho)(\theta \ell + (1 - \theta)v_{t+1}). \quad (3)$$

The terms are determined as follows. With probability ρ , B's investment pays off and B repays R . With probability $(1 - \rho)\theta$, B's investment does not payoff and the debtholder H_t is hit by a liquidity shock. Since the loan is demandable, but not tradeable, H_t redeems on demand and gets ℓ . With probability $(1 - \rho)(1 - \theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains B's debt at Date $t + 1$, which has value v_{t+1} at Date t since there is no discounting.

By stationarity ($v_t = v_{t+1} \equiv v$), equation (3) gives

$$v = \frac{\rho R + (1 - \rho)\theta \ell}{\rho + (1 - \rho)\theta}. \quad (4)$$

Given the horizon mismatch assumption (\star), the expression above is always less than the cost of investment c . Thus, if B borrows via a puttable loan, B cannot satisfy its borrowing constraint ($\$$) and therefore cannot invest.

LEMMA 3. *If B borrows via a puttable loan, it cannot fund its investment.*

¹⁵Formally, the value of holding B's debt is the Date- t *expected* value of B's debt at Date $t + 1$, i.e. we should write $\mathbb{E}_t[v_{t+1}]$ instead of v_{t+1} . For now, we focus on deterministic equilibria. Thus, this difference is immaterial and we omit the expectation operator for simplicity. (In Subsection 3.7, we do keep track of the expectation operator.)

Note that the value of the puttable loan (equation (4)) is greater than the value of the standard loan (equation (2)). Thus, demandability (i.e. “puttability”) is adding value and loosening B’s borrowing constraint. This is because it offers C_0 partial insurance against liquidity shocks: if C_0 is hit by a liquidity shock, he cannot necessarily get the full repayment R , but he can still demand redemption and at least get ℓ . This liquidity insurance may rationalize demandability in some circumstances. Indeed, it is reminiscent of the rationale for demandable debt in Calomiris and Kahn (1991), in that inefficient liquidation on the equilibrium path insures the creditor against bad outcomes.¹⁶ In our setting, however, the horizon mismatch is so severe that early liquidation is relatively likely. Thus, it is too expensive for B to insure C_0 by liquidating its investment whenever C_0 needs liquidity. If B makes its debt tradeable, however, the secondary debt market provides C_0 with insurance, even without liquidation. We turn to this next.

3.4 Bond

Now we consider a bond, i.e. tradeable long-term debt. At Date t , the value v_t of the bond can be written recursively:

$$v_t = \rho R + (1 - \rho) \left(\theta \sigma_t p_t + (1 - \theta) v_{t+1} \right). \quad (5)$$

The terms are determined as follows. With probability ρ , B’s investment pays off and B repays R . With probability $(1 - \rho)\theta$, B’s investment does not payoff and the debtholder H_t is hit by a liquidity shock. Since the bond is tradeable, but not demandable, H_t gets p_t if he finds a counterparty, which happens with probability σ_t , and nothing otherwise. With probability $(1 - \rho)(1 - \theta)$, B’s investment does not pay off and H_t is not hit by a liquidity shock. H_t retains B’s debt at Date $t + 1$, which has value v_{t+1} at Date t since there is no discounting.

To solve for the value v_t , we must first give the secondary-market price of the bond p_t .

LEMMA 4. *The secondary-market price of the bond is $p_t = v_t/2$.*

The bond price splits the gains from trade fifty-fifty between H_t and C_t , given that H_t is hit by a liquidity shock. Since H_t has value zero in this case (H_t dies at the end of the period and the bond is not demandable), the gains from trade are just the value v_t of the bond to the new debtholder C_t .

¹⁶In Calomiris and Kahn (1991), “bad outcomes” are associated with moral hazard problems, rather than liquidity shocks.

By stationarity ($v_t = v_{t+1} \equiv v$ and $\sigma_t \equiv \sigma$), the preceding lemma implies $p_t \equiv p \equiv v/2$, so equation (5) gives

$$v = \frac{\rho R}{\rho + (1 - \rho)\theta(1 - \sigma/2)}. \quad (6)$$

Given the horizon mismatch assumption (\star), the expression above is always less than the cost of investment c , even if the bond circulates (i.e. $\sigma = 1$). Thus, if B borrows via a bond, B cannot satisfy its borrowing constraint ($\$$) and therefore cannot invest.

LEMMA 5. *If B borrows via a bond, it cannot fund its investment.*

As in the case of the puttable loan above, the value of the bond (equation (6)) is greater than the value of the standard loan (equation (2)). Thus, tradeability is adding value, loosening B's borrowing constraint. This is because it offers C_0 partial insurance against liquidity shocks: if C_0 is hit by a liquidity shock, it cannot necessarily get the full repayment R , but it can still sell the bond in the secondary market and at least get $p = v/2$. This is the first step in showing that tradeability in the secondary market, or "market liquidity," helps loosen borrowing constraints in the primary market, or improves "funding liquidity." However, trading frictions in the OTC market depress the secondary-market bond price ($p \ll v$). Thus, despite tradeability, liquidity shocks are costly for creditors and ultimately tradeability alone is not enough for B to get its investment up and running.

3.5 Banknote

Now we consider a banknote, i.e. tradeable, demandable debt. At Date t , the value v_t of the banknote can be written recursively:

$$v_t = \rho R + (1 - \rho)\left(\theta(\sigma_t p_t + (1 - \sigma_t)\ell) + (1 - \theta)v_{t+1}\right). \quad (7)$$

The terms are determined as follows. With probability ρ , B's investment pays off and B repays R . With probability $(1 - \rho)\theta$, B's investment does not payoff and the debtholder H_t is hit by a liquidity shock. Since the banknote is both tradeable and demandable, H_t gets p_t if he finds a counterparty, which happens with probability σ_t , and otherwise redeems on demand and gets ℓ . With probability $(1 - \rho)(1 - \theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains the banknote at Date $t + 1$, which has value v_{t+1} at Date t since there is no discounting.

To solve for the value v_t , we must first give the secondary-market price of the banknote p_t .

LEMMA 6. *The secondary-market price of the banknote is $p_t = (v_{t+1} + \ell)/2$.*

The price of the banknote splits the gains from trade fifty-fifty between H_t and C_t , given that H_t is hit by a liquidity shock. Since H_t has value ℓ (H_t redeems on demand and gets ℓ if he does not trade with C_t), the gains from trade are $v_t - \ell$, the value to the new debtholder C_t minus the value to the current debtholder H_t . The price that splits these gains is $p_t = \ell + (v_t - \ell)/2 = (v_t + \ell)/2$. Critically, the secondary-market price of the banknote is higher than the secondary-market price of the bond. This is because the option to redeem on demand improves H_t 's bargaining position, since H_t gets a higher payoff if bargaining breaks down. Even if no debtholder ever demands redemption from B in the primary market, the option to do so can have important implications for the price of debt in the OTC secondary market.

This result suggests that the more costly it is to liquidate (the lower is ℓ), the lower is the secondary-market price p_t . This may cast light on the fact that nineteenth-century banknotes traded at a discount in markets far from the issuing bank, and that this discount was increasing in the distance to the issuer (Gorton (1996)): the farther you were from the issuer, the costlier it was for you to liquidate on demand and thus the weaker your bargaining position in the OTC market.

By stationarity ($v_t = v_{t+1} \equiv v$ and $\sigma_t \equiv \sigma$), the preceding lemma implies that $p_t \equiv p \equiv (v + \ell)/2$, so equation (7) gives

$$v = \frac{\rho R + (1 - \rho)\theta(1 - \sigma/2)\ell}{\rho + (1 - \rho)\theta(1 - \sigma/2)}. \quad (8)$$

Despite the horizon mismatch assumption (\star), the expression above may be greater than the cost of investment c . Thus, if B borrows via a banknote, B may be able to satisfy his borrowing constraint ($\$$) and therefore invest.

PROPOSITION 1. (DEMANDABILITY INCREASES DEBT CAPACITY.) *Suppose that*

$$\frac{1}{\theta} \frac{2(y - c)}{(c - \ell)(1 - \rho)} \geq \frac{1}{\rho}. \quad (9)$$

If B borrows via a banknote and the banknote circulates ($\sigma = 1$), then B can fund its investment.

B can increase its debt capacity by borrowing via demandable debt, even if it is never redeemed on demand on the equilibrium path. This proposition thus suggests a new rationale for demandable debt: it props up the price in secondary markets and thereby loosens borrowing constraints in primary markets. Demandability increases the market liquidity of B's debt, which provides creditors with insurance against liquidity shocks in the future. This makes them more willing to provide B with funding liquidity today. Thus, market liquidity and funding liquidity are complements and, likewise, tradeability and demandability are complements.

3.6 Money Runs

Having established that B can borrow only with tradeable, demandable debt (banknotes), we now turn to secondary-market liquidity and the possibility that a debtholder demands early redemption. To do this, we assume that B has issued a banknote with promised repayment R and look at the equilibria of the subgames for $t > 0$. (We determine R in equilibrium in Proposition 3).

First, observe that B's banknote indeed circulates as long as $\sigma_t = 1$ is a best response to the belief that $C_{t'}$ plays $\sigma_{t'} = 1$ for all $t' > t$. This is the case as long as C_t is willing to pay the search cost k to gain the surplus $v - p$ given $\sigma = 1$, or

$$k \leq v - p \Big|_{\sigma=1} = \frac{\rho(R - \ell)}{2\rho + (1 - \rho)\theta}, \quad (10)$$

having substituted in from Lemma 6 and equation (8).

But there may also be another equilibrium in which B's banknote does not circulate. B's banknote does not circulate as long as $\sigma_t = 0$ is a best response to the belief that $C_{t'}$ plays $\sigma_{t'} = 0$ for all $t' > t$. This is the case as long as C_t is *not* willing to pay the search cost k to gain the surplus $v - p$ given $\sigma = 0$, or

$$k \geq v - p \Big|_{\sigma=0} = \frac{1}{2} \frac{\rho(R - \ell)}{\rho + (1 - \rho)\theta}, \quad (11)$$

again having substituted in from Lemma 6 and equation (8).

PROPOSITION 2. (MONEY RUNS.) *Suppose that B borrows via a banknote with promised repayment R and the search cost k is such that*

$$\frac{1}{2} \frac{\rho(R - \ell)}{\rho + (1 - \rho)\theta} \leq k \leq \frac{\rho(R - \ell)}{2\rho + (1 - \rho)\theta}. \quad (12)$$

The $t > 0$ subgame has both an equilibrium in which B's debt circulates ($\sigma = 1$) and there is no early liquidation and an equilibrium in which B's debt does not circulate ($\sigma = 0$) and there is early liquidation.

Thus demandable debt has a dark side: if a counterparty C_t doubts future liquidity, i.e. he doubts that he will find a counterparty in the future, then C_t will not search. As a result, the debtholder H_t indeed will not find a counterparty. There is a self-fulfilling dry-up of secondary-market liquidity. With demandable debt, this has severe real effects: unable to trade, H_t redeems its debt on demand, leading to costly liquidation of B's investment. In other words, a change in just the beliefs about future liquidity leads to the failure of B's debt as a medium of exchange in the secondary market—the failure of B's debt as money. As a result, there is sudden withdrawal of liquidity from B, i.e. a bank run, or a *money run*.

COROLLARY 1. *Suppose k satisfies condition (12). If C_t 's beliefs change from $\sigma_{t'} = 1$ to $\sigma_{t'} = 0$ for $t' > t$, the debtholder H_t “runs” on B , i.e. H_t unexpectedly demands redemption of his debt, forcing B to liquidate its investment.*

Demandability cuts both ways in the secondary market. It increases B 's debt capacity by creating market liquidity, propping up the price of B 's debt. But it also exposes B to money runs, since B must provide liquidity on demand if market liquidity dries up. Thus, financial fragility may be a necessary evil, resulting from the need to overcome funding constraints given a horizon mismatch between the investment and the creditors' liquidity needs.

3.7 Equilibrium Runs

We now turn to characterizing an equilibrium in which B borrows via a banknote and money runs arise on the equilibrium path. To do this, we expand the model slightly to introduce a “sunspot” coordination variable at each date, $s_t \in \{0, 1\}$. We will interpret $s_t = 1$ as “normal times” and $s_t = 0$ as a “confidence crisis,” since the sunspot does not affect economic fundamentals, but serves only as a way for agents to coordinate their beliefs. We assume that $s_0 = 1$, that $\mathbb{P}[s_{t+1} = 0 | s_t = 1] =: \lambda$, and that $\mathbb{P}[s_{t+1} = 0 | s_t = 0] = 1$, where we think about λ as a small number. In words: the economy starts in normal times and a permanent confidence crisis occurs randomly with small probability λ .

We now look for a Markov equilibrium, i.e. an equilibrium in which the sunspot (rather than the whole history) is a sufficient statistic for C_t 's action:

$$\sigma_t = \begin{cases} \sigma^1 & \text{if } s_t = 1, \\ \sigma^0 & \text{if } s_t = 0. \end{cases} \quad (13)$$

Note that the baseline case of a stationary equilibrium is the special case of $\lambda = 0$. We can now write the banknote's value v^0 when $s_t = 0$ and v^1 when $s_t = 1$ (cf. the analogous equation for the stationary case in equation (7)):

$$v^0 = \rho R + (1 - \rho) \left(\theta \left(\sigma^0 p^0 + (1 - \sigma^0) \ell \right) + (1 - \theta) v^0 \right), \quad (14)$$

$$v^1 = \rho R + (1 - \rho) \left(\theta \left(\sigma^1 p^1 + (1 - \sigma^1) \ell \right) + (1 - \theta) \left(\lambda v^0 + (1 - \lambda) v^1 \right) \right). \quad (15)$$

The next proposition characterizes an equilibrium in which the “confidence crisis” induces a money run.

PROPOSITION 3. (EQUILIBRIUM WITH SUNSPOT RUNS.) *Suppose that the condition in*

equation (9) is satisfied. As long as λ is sufficiently small, there exists k such that B can fund its investment only with tradeable, demandable debt (a banknote), even though it admits a money run when $s_t = 0$. Specifically, C_t plays $\sigma_t = s_t$, and the value of the banknote when $s_t = 0$ is

$$v^0 = \frac{\rho R + (1 - \rho)\theta\ell}{\rho + (1 - \rho)\theta} \quad (16)$$

the value of the banknote when $s_t = 1$ is

$$v^1 = \frac{\rho R + (1 - \rho)\left(\theta\ell/2 + (1 - \theta/2)\lambda v_0\right)}{\rho + (1 - \rho)\left(\theta/2 + (1 - \theta/2)\lambda\right)}, \quad (17)$$

and the promised repayment is

$$R = y - \frac{(\rho + (1 - \rho)\theta)(\rho + (1 - \rho)\lambda)}{\rho(\rho + (1 - \rho)(\theta + (1 - \theta)\lambda))}(v^1 - c). \quad (18)$$

We have written the system recursively, expressing v^1 as a function of v^0 and R as a function of v^1 , for simplicity. We give a closed-form expression for R in terms of primitives in Appendix 3.

3.8 Circulating Bonds and Liquidity Dry-ups without Runs

So far, we have maintained the horizon mismatch assumption (\star), which implied that only the banknote was feasible. In this section, we relax this assumption, so that bonds are feasible too. We find that bonds have two advantages over banknotes: (i) they are more liquid in the secondary market, which can help with financing, and (ii) they are not subject to runs, which can prevent inefficient liquidation.

PROPOSITION 4. (BOND FINANCE.) *Suppose that*

$$\frac{1}{\theta} \frac{y - c}{(c - \ell)(1 - \rho)} \leq \frac{1}{\rho} \leq \frac{1}{\theta} \frac{2(y - c)}{c(1 - \rho)}, \quad (19)$$

and

$$\frac{\rho(y - \ell)}{2\rho + (1 - \rho)\theta} \leq k \leq \frac{\rho y}{2\rho + (1 - \rho)\theta}. \quad (20)$$

B can fund its investment only with tradeable, long-term debt (a bond).

Intuitively, this result says that if there is a horizon mismatch which is not too severe (condition (19)), then B can borrow via a bond, but cannot borrow via a non-circulating instrument, such as a puttable loan. And, further, if the search cost k is large enough (condition (20)), then B cannot borrow via a banknote either. The reason is that demandability increases secondary-market prices, which can discourage search: whereas

a counterparty may be willing to pay k to search for a bond, since he anticipates buying at a discount, he may be reluctant to pay k to search for a banknote, since he anticipates paying a high price. Thus, the banknote may not circulate—and a non-circulating banknote is effectively just a puttable loan.

Even though a counterparty is relatively willing to pay k to search for a bond, liquidity dry-ups can still occur in the secondary market for the bond.

COROLLARY 2. Suppose that B borrows via a bond with promised repayment R and that

$$\frac{1}{2} \frac{\rho R}{\rho + (1 - \rho)\theta} \leq k \leq \frac{\rho R}{2\rho + (1 - \rho)\theta}. \quad (21)$$

The $t > 0$ subgame has both an equilibrium in which B's debt circulates ($\sigma = 1$) and an equilibrium in which B's debt does not circulate ($\sigma = 0$). However, there is not early liquidation in either equilibrium; B's investment always pays off y .

Like trade in banknotes, trade in bonds is self-fulfilling. A self-fulfilling liquidity dry-up can arise whenever a counterparty searches if and only if he believes that other counterparties will search in the future (cf. equations (10) and (11) in Subsection 3.6). However, since the bond is not demandable, liquidity dry-ups in the secondary market do not lead to runs or inefficient liquidation of B's investment.

The analysis above captures the two advantages that bonds have over banknotes: (i) dry-ups in bond liquidity are “less likely” than dry-ups in banknote liquidity, since counterparties search for bonds for larger k , and (ii) they are less costly, since they do not lead to B's liquidation. Thus, we expect to see demandable debt mainly when the horizon mismatch is so severe that borrowing via bonds is impossible. We think this casts light on why bank debt is more likely to be demandable than corporate debt: the horizon mismatch assumption (\star) is more likely to hold for banks than for other firms.

4 Conclusion

One important function of banks is to create money, i.e. to issue debt that circulates in OTC markets. By focusing on this function of banks, we found a new type of bank runs—“money runs”—and a new rationale for demandable debt, both of which are the result of how bank debt circulates—or fails to circulate—in secondary markets. Money runs occur because secondary-market liquidity is fragile and self-fulfilling. Banks issue demandable debt because its secondary-market price is high. In contrast to the literature, our results suggest that financial fragility may be a necessary evil and, further, that regulating markets may help bank stability more than regulating banks themselves.

A Proofs

A.1 Proof of Lemma 1

The result follows immediately from C_0 's participation constraint: B and C_0 find an agreement point if and only if bargaining gives both more than their disagreement utilities. Since C_0 receives v_0 from bargaining, this must exceed its cost c . \square

A.2 Proof of Lemma 2

By Lemma 1, B can invest only if $v_0 = v \geq c$ or, by equation (2),

$$\frac{\rho R}{\rho + (1 - \rho)\theta} \geq c. \quad (22)$$

This says that

$$\frac{1}{\theta} \frac{R - c}{c(1 - \rho)} \geq \frac{1}{\rho} \quad (23)$$

which violates the assumption (\star) since $R \leq y$. \square

A.3 Proof of Lemma 3

By Lemma 1, B can invest only if $v_0 = v \geq c$ or, by equation (4),

$$\frac{\rho R + (1 - \rho)\theta\ell}{\rho + (1 - \rho)\theta} \geq c. \quad (24)$$

Since $\ell < c/2$, a necessary condition for this is that

$$\frac{\rho R + (1 - \rho)\theta c/2}{\rho + (1 - \rho)\theta} \geq c. \quad (25)$$

This says that

$$\frac{1}{\theta} \frac{2(R - c)}{c(1 - \rho)} \geq \frac{1}{\rho} \quad (26)$$

which violates the assumption (\star) since $R \leq y$. \square

A.4 Proof of Lemma 4

When C_t and H_t are matched H_t has been hit by a liquidity shock. Thus, C_t 's value of the bond is v_t and H_t 's value of the bond is zero (since H_t consumes only at Date t and the bond is not demandable). The total surplus is thus v_t , which C_t and H_t split fifty-fifty in accordance with the Nash bargaining solution. Thus the price is $p_t = v_t/2$.

A.5 Proof of Lemma 5

By Lemma 1, B can invest only if $v_0 = v \geq c$ or, by equation (6),

$$\frac{\rho R}{\rho + (1 - \rho)\theta(1 - \sigma/2)} \geq c. \quad (27)$$

This is increasing in σ , so a necessary condition is that the above holds for $\sigma = 1$, or

$$\frac{\rho R}{\rho + (1 - \rho)\theta/2} \geq c. \quad (28)$$

This says that

$$\frac{1}{\theta} \frac{2(R - c)}{c(1 - \rho)} \geq \frac{1}{\rho} \quad (29)$$

which violates the assumption (\star) since $R \leq y$. \square

A.6 Proof of Lemma 6

When C_t and H_t are matched H_t has been hit by a liquidity shock. Thus, C_t 's value of the banknote is v_t and H_t 's value of the banknote is ℓ (since H_t consumes only at Date t , it redeems on demand if it does not trade). The gains from trade are thus $v_t - \ell$, which C_t and H_t split fifty-fifty in accordance with the Nash bargaining solution, i.e. p_t is such that

$$H_t \text{ gets } \frac{1}{2}(v_t - \ell) + \ell = p_t, \quad (30)$$

$$C_t \text{ gets } \frac{1}{2}(v_t - \ell) = v_t - p_t, \quad (31)$$

or $p_t = (v_t + \ell)/2$.

A.7 Proof of Proposition 1

By Lemma 1, B can invest if $v_0 = v \geq c$ or, by equation (8) with $\sigma = 1$,

$$\frac{2\rho R + (1 - \rho)\theta\ell}{2\rho + (1 - \rho)\theta} \geq c. \quad (32)$$

This says that

$$\frac{1}{\theta} \frac{2(R - c)}{(c - \ell)(1 - \rho)} \geq \frac{1}{\rho}. \quad (33)$$

This is feasible for some $R \leq y$, i.e. as long as

$$\frac{1}{\theta} \frac{2(y - c)}{(c - \ell)(1 - \rho)} \geq \frac{1}{\rho}, \quad (34)$$

which is the condition in the proposition (note this is mutually compatible with the horizon mismatch assumption (\star)). \square

A.8 Proof of Proposition 2

The argument is in the text.

A.9 Proof of Corollary 1

The result follows immediately from Proposition 2.

A.10 Proof of Proposition 3

We first solve for the values v^0 and v^1 given the strategies $\sigma^0 = 0$ and $\sigma^1 = 1$. We then show that these strategies are indeed best responses (for some k). Finally, we compute the repayment R in accordance with the Nash bargaining solution.

Values. From equation (14) with $\sigma^0 = 0$, we have immediately that

$$v^0 = \frac{\rho R + (1 - \rho)\theta\ell}{\rho + (1 - \rho)\theta} \quad (35)$$

(this is just the value of the puttable loan in equation (4)). From Lemma 6 (the logic of which is not affected by the presence of sunspots), we have the price

$$p^1 = \frac{\lambda v^0 + (1 - \lambda)v^1 + \ell}{2}. \quad (36)$$

Thus, equation (15) with $\sigma^1 = 1$ reads

$$v^1 = \rho R + (1 - \rho) \left(\theta \frac{\lambda v^0 + (1 - \lambda)v^1 + \ell}{2} + (1 - \theta) (\lambda v^0 + (1 - \lambda)v^1) \right), \quad (37)$$

so

$$v^1 = \frac{\rho R + (1 - \rho) (\theta\ell/2 + (1 - \theta/2)\lambda v_0)}{\rho + (1 - \rho) (\theta/2 + (1 - \theta/2)\lambda)} \quad (38)$$

Best responses. $\sigma^1 = 1$ and $\sigma^0 = 0$ are best response if

$$v^0 - p^0 \leq k \leq v^1 - p^1 \quad (39)$$

or

$$v^0 - \frac{v^0 + \ell}{2} \leq k \leq v^1 - \frac{\lambda v^0 + (1 - \lambda)v^1 + \ell}{2}. \quad (40)$$

This is satisfied for some k as long as $v^1 \geq v^0$, which is the case as long as $R \geq \ell$, which must be the case.

Repayment. The repayment R is determined by Nash bargaining between B and C_0 . Now denote B's value in state s by u^s . Since the state is $s = 1$ at Date 0, B and C_0 agree on a repayment R such that B's value is u^1 and C_0 's value is v^1 . The gains from trade are thus $u^1 + v^1 - c$. R is thus such that

$$\text{B gets } \frac{1}{2}(u^1 + v^1 - c) = u^1, \quad (41)$$

$$\text{C}_0 \text{ gets } \frac{1}{2}(u^1 + v^1 - c) + c = v^1, \quad (42)$$

so $u^1 = v^1 - c$.

From the expressions above we can compute R in terms of primitives. First we compute B's value u^0 in state 0. In this case, the banknote does not circulate and the banknote is effectively like the puttable loan in Subsection 3.3. Whenever the creditor is hit by a liquidity shock, he redeems on demand and B liquidates and gets zero. B's value is thus

$$u^0 = \rho(y - R) + (1 - \rho)(1 - \theta)u^0 \quad (43)$$

or

$$u^0 = \frac{\rho(y - R)}{\rho + (1 - \rho)\theta}. \quad (44)$$

Now, B's value in state 1 is given by

$$u^1 = \rho(y - R) + (1 - \rho)\left((1 - \lambda)u^1 + \lambda(1 - \theta)u^0\right) \quad (45)$$

or, substituting in for u^0 from equation (44) above,

$$u^1 = \frac{\rho(y - R)}{\rho + (1 - \rho)\lambda} \left(1 + \frac{(1 - \rho)\lambda(1 - \theta)}{\rho + (1 - \rho)\theta}\right). \quad (46)$$

Substituting for this into the bargaining outcome $u^1 = v^1 - c$ gives the expression for the repayment R in terms of v_1 as stated in the proposition.

Now we can also substitute for v^1 from equation (38) and v_0 from equation (35) above to get

$$R = \frac{Ay + Bc - C\ell}{D} \quad (47)$$

where the constants A , B , C , and D are defined as follows:

$$\begin{aligned} A &= \left(2(\rho + (1 - \rho)\lambda + (1 - \rho)(1 - \lambda)\theta)\right) \left((\rho + (1 - \rho)\theta)(\rho + (1 - \rho)\lambda) - (1 - \rho)\theta\lambda\right), \\ B &= \left(2(\rho + (1 - \rho)\lambda + (1 - \rho)(1 - \lambda)\theta)\right) (\rho + (1 - \rho)\theta)(\rho + (1 - \rho)\lambda), \\ C &= (\rho + (1 - \rho)\lambda)(1 - \rho) \left(\rho + (1 - \rho)(2\lambda + (1 - \lambda)\theta)\right), \\ D &= \rho \left(\theta(5 + 4\lambda)(1 - \rho)(\rho + (1 - \rho)\lambda) + \theta^2(1 - \lambda)^2(1 - \rho)^2 + 4(\rho + (1 - \rho)\lambda)^2\right). \end{aligned}$$

A.11 Proof of Proposition 4

We first show that for k satisfying condition (20), the bond can circulate but the banknote cannot. We then show that if the bond circulates, B can fund its investment with a bond. However, if the banknote does not circulate, B cannot fund its investment with a banknote. This also implies that B cannot fund its investment with a puttable loan or a loan: a puttable loan is identical to a non-circulating banknote, which is infeasible, and a puttable loan always has higher debt capacity than a loan (cf. equations (2) and (4)).

Bond can circulate. The bond circulates whenever C_t 's value v less the price p exceeds his cost of searching k , or, given the value in equation (6) and the price in Lemma 4,

$$\frac{1}{2} \frac{\rho R}{\rho + (1 - \rho)\theta(1 - \sigma/2)} \geq k. \quad (48)$$

For $R = y$ and $\sigma = 1$ this reads

$$\frac{\rho y}{2\rho + (1 - \rho)\theta} \geq k, \quad (49)$$

which is satisfied by assumption (condition (20)). Thus, the bond can circulate as long as R is sufficiently large.

Banknote cannot circulate. The banknote circulates whenever C_t 's value v less the price p exceeds his cost of searching k , or, given the value in equation (8) and the price in Lemma 6,

$$\frac{1}{2} \frac{\rho(R - \ell)}{\rho + (1 - \rho)\theta(1 - \sigma/2)} \geq k. \quad (50)$$

The left-hand side is maximized when $R = y$ and $\sigma = 1$, for which the inequality reads

$$\frac{\rho(y - \ell)}{2\rho + (1 - \rho)\theta} \geq k, \quad (51)$$

which is never satisfied by assumption (condition (20)). Thus, the banknote cannot circulate.

Bond financing feasible. Given the bond circulates ($\sigma = 1$), the borrowing

constraint (\$) is satisfied whenever

$$\frac{\rho R}{\rho + (1 - \rho)\theta/2} \geq c, \quad (52)$$

given the value of the bond from equation (6). This can be re-written as

$$\frac{1}{\rho} \leq \frac{1}{\theta} \frac{2(R - c)}{c(1 - \rho)}. \quad (53)$$

This is satisfied by assumption (condition (19)) as long as R is sufficiently large.

Banknote financing infeasible. Given the banknote does not circulate ($\sigma = 0$), the borrowing constraint (\$) is satisfied whenever

$$\frac{\rho R + (1 - \rho)\theta\ell}{\rho + (1 - \rho)\theta} \geq c, \quad (54)$$

given the value of the banknote from equation (8). This can be re-written as

$$\frac{1}{\rho} \leq \frac{1}{\theta} \frac{R - c}{(c - \ell)(1 - \rho)}. \quad (55)$$

The right-hand side is maximized when $R = y$, for which the inequality reads

$$\frac{1}{\rho} \leq \frac{1}{\theta} \frac{y - c}{(c - \ell)(1 - \rho)}. \quad (56)$$

This is never satisfied by assumption (condition (19)). □

A.12 Proof of Corollary 2

The argument here is analogous to the argument for the banknote in Subsection 3.6: there are multiple equilibria whenever (i) a counterparty's searching is a best response other counterparties' searching and (ii) a counterparty's not searching is a best response to other counterparties' not searching, or

$$v - p \Big|_{\sigma=0} \leq k \leq v - p \Big|_{\sigma=1}. \quad (57)$$

Substituting in for the value of the bond v from equation (6) and the price of the bond p from Lemma 4, this reads

$$\frac{1}{2} \frac{\rho R}{\rho + (1 - \rho)\theta} \leq k \leq \frac{\rho R}{2\rho + (1 - \rho)\theta}, \quad (58)$$

as stated in the corollary. □

B Table of Notations

Players and Indices	
t	time index
B	borrower or “bank”
C_t	(potential) creditor/counterparty at Date t
H_t	debtholder at Date t
Technologies and Preferences	
y	payoff of B’s investment
c	cost of B’s investment
ℓ	liquidation value of B’s investment
ρ	probability B’s investment pays off each date
θ	probability with which creditor is hit by liquidity shock at each date
Prices, Values, and Strategies	
R	B’s promised repayment (face value of debt)
v_t	value of B’s debt to a creditor at Date t
p_t	secondary-market price of B’s debt at Date t
σ_t	mixed strategy of counterparty C_t
Other Variables	
s_t	sunspot at Date t (Subsection 3.7)
λ	$\mathbb{P}[s_{t+1} = 0 s_t = 1]$, “confidence crisis” probability (Subsection 3.7)

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